Math 220 Sample Midterm 1

Name: _____

Recitation instructor: _____

Recitation time: _____

- This is a closed-book, closed-notes exam. No calculators or electronic aids are permitted. Please make sure that your cell phone is turned off.

- Read each question carefully and show your work.

- You will have 75 minutes to complete the exam.

Problem 1. Evaluate the following limits.

A. $\lim\limits_{x \to 2} (x^3 + 2x + 1)$

B. $\lim\limits_{\theta \to \pi/2} \dfrac{\cos(\theta)}{\theta}$

C. $\lim\limits_{\theta \to 0} \dfrac{5(1 - \cos\theta)}{\theta}$

Problem 2.

A. $\lim\limits_{t \to 1} \dfrac{t^2 + t - 2}{t - 1}$

B. $\lim\limits_{x \to 7} \dfrac{\sqrt{x + 2} - 3}{x - 7}$

Problem 3. Given that $\lim\limits_{x\to 5} u(x) = 8$ and $\lim\limits_{x\to 5} w(x) = 2$, find the following limits.

A. $\lim\limits_{x\to 5} \dfrac{w(x)^2 - 9}{u(x)}$

B. $\lim\limits_{x\to 5} \dfrac{\sqrt{u(x) \cdot w(x)}}{x + 5}$
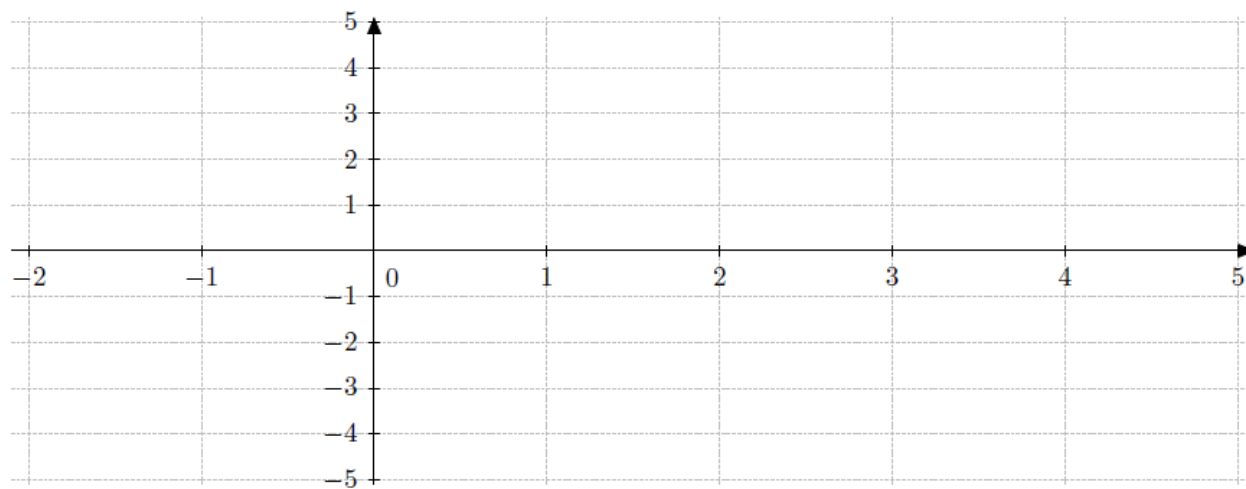
Problem 4. Use the squeeze theorem to find

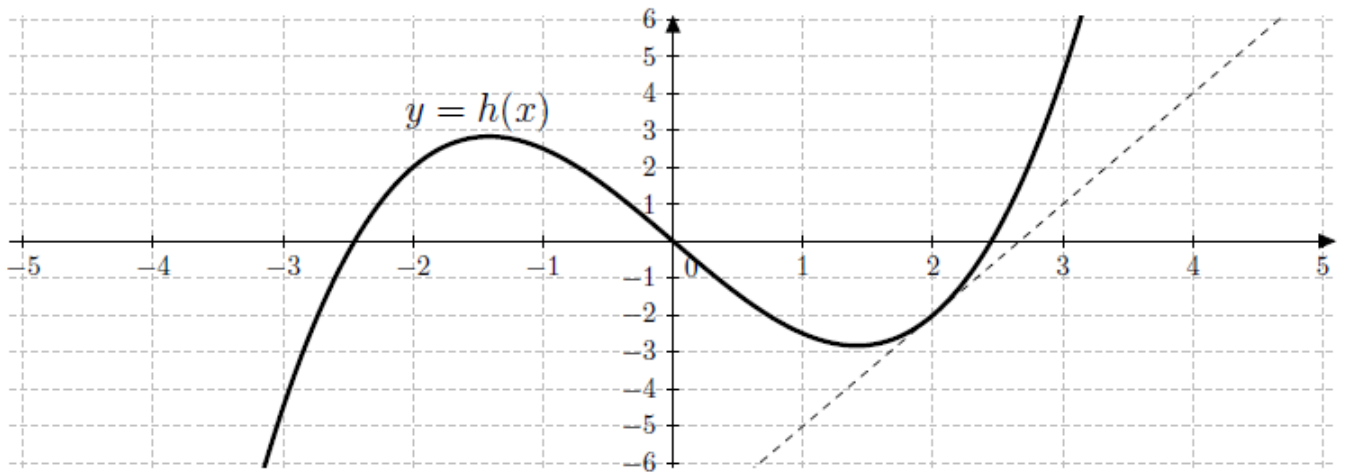$$\lim_{x\to 0} x \sin\left(\frac{1}{x^2}\right)$$

Problem 5. Let
$$f(x) = \begin{cases} x^2 + 1 & \text{if } x \neq 1 \\ 6 & \text{if } x = 1. \end{cases}$$
Where is $f(x)$ continuous/discontinuous?

Problem 6. Sketch the graph of a function $k(x)$ that satisfies $\lim\limits_{x \to 0} k(x) = 2$, $\lim\limits_{x \to 4^-} k(x) = -2$, $\lim\limits_{x \to 4^+} k(x) = 3$, and $k(4) = 1$.

Problem 7.

The function $y = h(x)$ is graphed above in solid bold. There is also a dotted line graphed. Find the following two values. [Answers are enough. No explanation is needed.]

A. $h(2) =$

B. $h'(2) =$

Problem 8. Suppose that an object is at position $s(t) = t^2 + 3$ feet at time t seconds.

A. Find the average velocity of the object over a time interval from time 3 seconds to time $3 + h$ seconds.
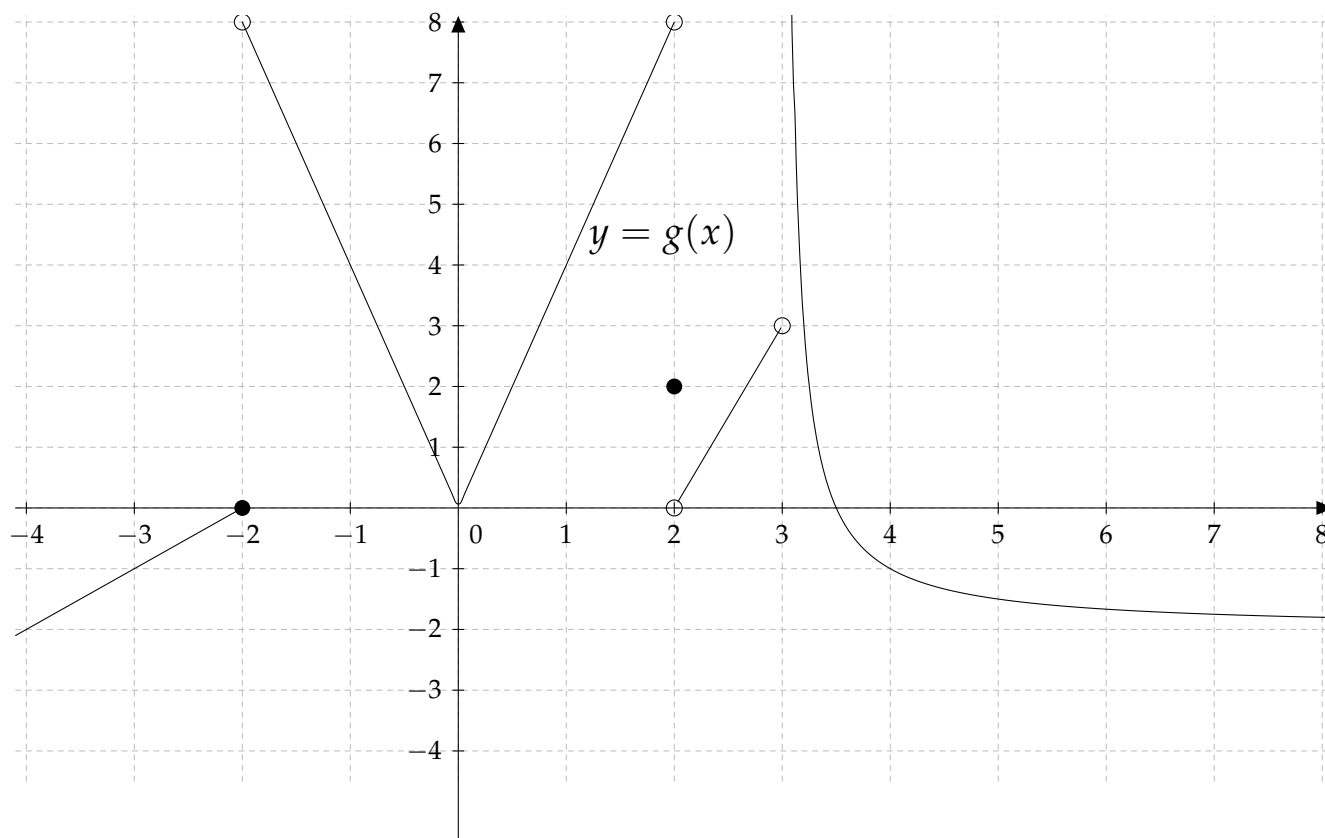
B. Find the instantaneous velocity of the object at time 3 seconds by taking the limit of the average velocity in Part A as $h \to 0$.

Problem 9. Let $v(x) = \dfrac{2}{x}$.

A. Find $v'(1)$ by using one of the limit definitions of the derivative.

B. Find the equation of the tangent line to $y = v(x)$ at $x = 1$.

Problem 10.

$y = g(x)$

State the value of each of the below quantities. If the quantity does not exist, write "does not exist" or "DNE". (Answers are enough. No explanation is needed.)

A. $\lim_{x \to -2^-} g(x) =$

B. $\lim_{x \to -2^+} g(x) =$

C. $\lim_{x \to -2} g(x) =$

D. $\lim_{x \to 3^+} g(x) =$

E. $\lim_{x \to 3^-} g(x) =$

F. $g'(1) =$

8

G. List all discontinuities and classify them as removable, infinite or jump