Math 220 Midterm 1

Name: ________________________________

Recitation instructor: ________________________________

Recitation time: ________________________________

- This is a closed-book, closed-notes exam. No calculators or electronic aids are permitted. Please make sure that your cell phone is turned off.
- Read each question carefully and show your work.
- You will have 75 minutes to complete the exam.

Grading

1	/9	**6**	/4
2	/6	**7**	/10
3	/5	**8**	/10
4	/6	**9**	/10
5	/5	**10**	/15
	/	**Total**	/80

Problem 1. (*9 points*) Evaluate the following limits.

A. (*3 points*) $\lim_{x \to -1} \frac{x^2 + x + 3}{x - 4}$ $= \frac{(-1)^2 + (-1) + 3}{-1 - 4} = \frac{3}{-5}$

B. (*3 points*) $\lim_{x \to 1} [\ln(x + 3) - 5x]$ $= \ln(1+3) - 5(1)$

$= \ln(4) - 5$

C. (*3 points*) $\lim_{\theta \to 0} \frac{4 \sin \theta}{\theta}$ $= 4 \lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 4(1) = 4$

Problem 2. (*6 points*) Let

$$f(x) = \begin{cases} e^x + 1 & \text{if } x \neq 0 \\ 2 & \text{if } x = 0. \end{cases}$$

Where is $f(x)$ continuous/discontinuous?

When $x \neq 0$, $f(x) = e^x + 1$ is continuous.

When $x = 0$, we consider the followings:

① $f(0) = 2$

② $\lim_{x \to 0} f(x) = \lim_{x \to 0} e^x + 1 = e^0 + 1 = 2$

③ $\lim_{x \to 0} f(x) = f(0)$.

So $f(x)$ is continuous at $x = 0$.

Problem 3. (*5 points*) Sketch the graph of a function $k(x)$ that satisfies $\lim_{x\to 1} k(x) = 3$, $\lim_{x\to 3^-} k(x) = 1$, $\lim_{x\to 3^+} k(x) = -3$, and $k(3) = -5$.

Problem 4. (*6 points*) Given that $\lim_{x\to 2} u(x) = 4$ and $\lim_{x\to 4} w(x) = 3$, find the following limits.

2

A. (*3 points*) $\lim_{x\to 2} \dfrac{w(x)^2+1}{\sqrt{u(x)}}$ $= \dfrac{3^2+1}{\sqrt{4}} = \dfrac{10}{2} = 5$

B. (*3 points*) $\lim_{x\to 2} \dfrac{x^2}{u(x)+3w(x)}$ $= \dfrac{2^2}{4+3(3)} = \dfrac{4}{13}$

Problem 5. (*5 points*) Use the squeeze theorem to find

$$\lim_{x\to 2} g(x)$$

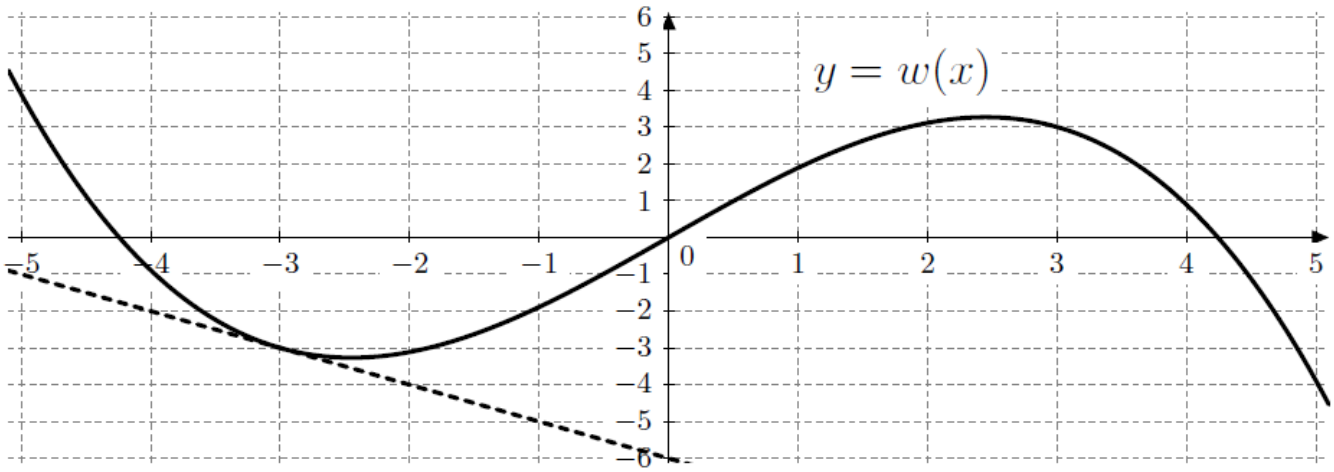
provided that the function $w(x)$ satisfies $3x+1 \le g(x) \le x^2+3$ for all $x \neq 2$.

$\lim_{x\to 2} 3x+1 = 3(2)+1 = 7$

$\lim_{x\to 2} x^2+3 = 2^2+3 = 7$

So $\lim_{x\to 2} g(x) = 7$

Problem 6. (*4 points*)



The function $y = w(x)$ is graphed above in solid bold. There is also a dotted line graphed. Find the following two values. [Answers are enough. No explanation is needed.]

A. (*2 points*) $w(-3) =$ -3

B. (*2 points*) $w'(-3) =$ -1

Problem 7. (*10 points*)

A. (*5 points*) $\lim_{t\to-3} \frac{t^2-2t-15}{t+3}$ $= \lim_{t\to-3} \frac{(t-5)(t+3)}{t+3}$

$= \lim_{t\to-3} t-5 = -3-5 = \boxed{-8}$

B. (*5 points*) $\lim_{x\to5} \frac{1-\sqrt{x-4}}{x-5}$ $= \lim_{x\to5} \frac{1-\sqrt{x-4}}{x-5} \cdot \frac{1+\sqrt{x-4}}{1+\sqrt{x-4}}$

$= \lim_{x\to5} \frac{1-(x-4)}{(x-5)(1+\sqrt{x-4})} = \lim_{x\to5} \frac{5-x}{(x-5)(1+\sqrt{x-4})}$

$= \lim_{x\to5} \frac{-1}{1+\sqrt{x-4}} = \frac{-1}{1+\sqrt{5-4}} = -\frac{1}{2}$

Problem 8. (*10 points*) Suppose that an object is at position $s(t) = 2t^2$ feet at time t seconds.

A. (*3 points*) Find the average velocity of the object over a time interval from time 1 seconds to time $1+h$ seconds.

Avg velocity $= \dfrac{2(1+h)^2 - 2}{h}$ ft/sec.

students don't need to simplify, but it's okay if they do.

B. (*7 points*) Find the instantaneous velocity of the object at time 1 second by taking the limit of the average velocity in Part A as $h \to 0$.

Inst velocity $= \lim_{h \to 0} \dfrac{2(1+h)^2 - 2}{h}$

$= \lim_{h \to 0} \dfrac{2(1+2h+h^2) - 2}{h}$

$= \lim_{h \to 0} \dfrac{2 + 4h + 2h^2 - 2}{h}$

$= \lim_{h \to 0} \dfrac{4h + 2h^2}{h} = \lim_{h \to 0} 4 + 2h$

$= 4$ ft/sec.

Problem 9. (*10 points*) Let $v(x) = \dfrac{4}{x^2}$.

A. (*7 points*) Find $v'(2)$ by using one of the limit definitions of the derivative.

$$v'(2) = \lim_{x \to 2} \frac{\frac{4}{x^2} - \frac{4}{2^2}}{x-2}$$

$$= \lim_{x \to 2} \frac{1}{x-2}\left[\frac{4}{x^2} - 1\right] = \lim_{x \to 2}\left[\frac{4-x^2}{x^2(x-2)}\right]$$

$$= \lim_{x \to 2} \frac{\cancel{(2-x)}(2+x)}{x^2\cancel{(x-2)}} = \lim_{x \to 2} \frac{-(2+x)}{x^2}$$

$$= -\frac{(2+2)}{2^2} = -1$$

B. (*3 points*) Find the equation of the tangent line to $y = v(x)$ at $x = 2$.

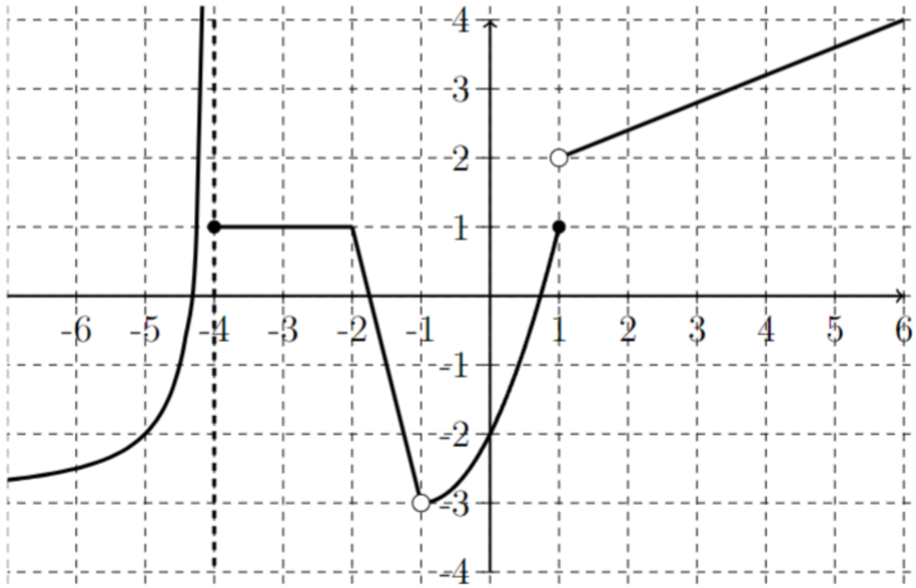
$a = 2$, $b = v(2) = \frac{4}{2^2} = 1$, $m = -1$

The equation is

$$\boxed{y - 1 = -1(x-2)}$$

students do not need to simplify.

Problem 10. (*15 points*)



Consider the graph $y = g(x)$ above. State the value of each of the below quantities (A - F: 2 points for each). If the quantity does not exist, write "does not exist" or "DNE". (Answers are enough. No explanation is needed.)

A. $\lim_{x \to -4^-} g(x) =$ ∞

B. $\lim_{x \to -4^+} g(x) =$ 1

C. $\lim_{x \to 1} g(x) =$ DNE

D. $\lim_{x \to -1} g(x) =$ -3

E. $\lim_{x \to -2} g(x) =$ 1

F. $g'(-3) =$ 0

G. (*3 points*) List all discontinuities and classify them as removable, infinite or jump.

-4, infinite

-1, removable

1, jump