

Math 220 Midterm 1

Name: _____

Recitation instructor: _____

Recitation time: _____

- This is a closed-book, closed-notes exam. No calculators or electronic aids are permitted. Please make sure that your cell phone is turned off.
- Read each question carefully and show your work.
- You will have 75 minutes to complete the exam.

Grading

1	/9	6	/4
2	/6	7	/10
3	/5	8	/10
4	/6	9	/10
5	/5	10	/15
	/	Total	/80

Problem 1. (9 points) Evaluate the following limits.

A. (3 points) $\lim_{x \rightarrow -1} \frac{x^2 + x + 3}{x - 4}$

B. (3 points) $\lim_{x \rightarrow 1} [\ln(x + 3) - 5x]$

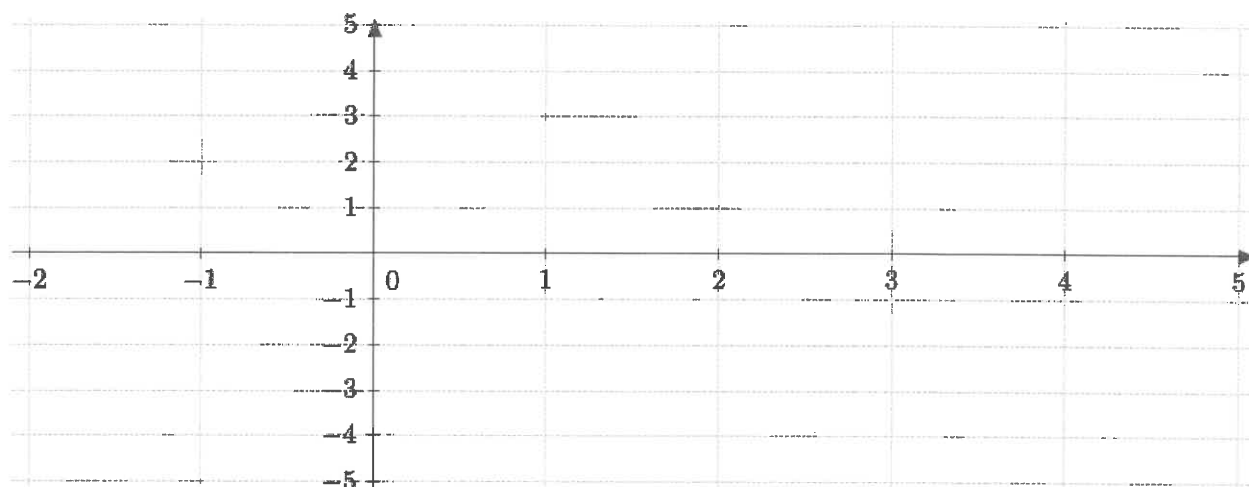
C. (3 points) $\lim_{\theta \rightarrow 0} \frac{4 \sin \theta}{\theta}$

Problem 2. (6 points) Let

$$f(x) = \begin{cases} e^x + 1 & \text{if } x \neq 0 \\ 2 & \text{if } x = 0. \end{cases}$$

Where is $f(x)$ continuous/discontinuous?

Problem 3. (5 points) Sketch the graph of a function $k(x)$ that satisfies $\lim_{x \rightarrow 1} k(x) = 3$, $\lim_{x \rightarrow 3^-} k(x) = 1$, $\lim_{x \rightarrow 3^+} k(x) = -3$, and $k(3) = -5$.



Problem 4. (6 points) Given that $\lim_{x \rightarrow 2} u(x) = 4$ and $\lim_{x \rightarrow 4} w(x) = 3$, find the following limits.

A. (3 points) $\lim_{x \rightarrow 2} \frac{w(x)^2 + 1}{\sqrt{u(x)}}$

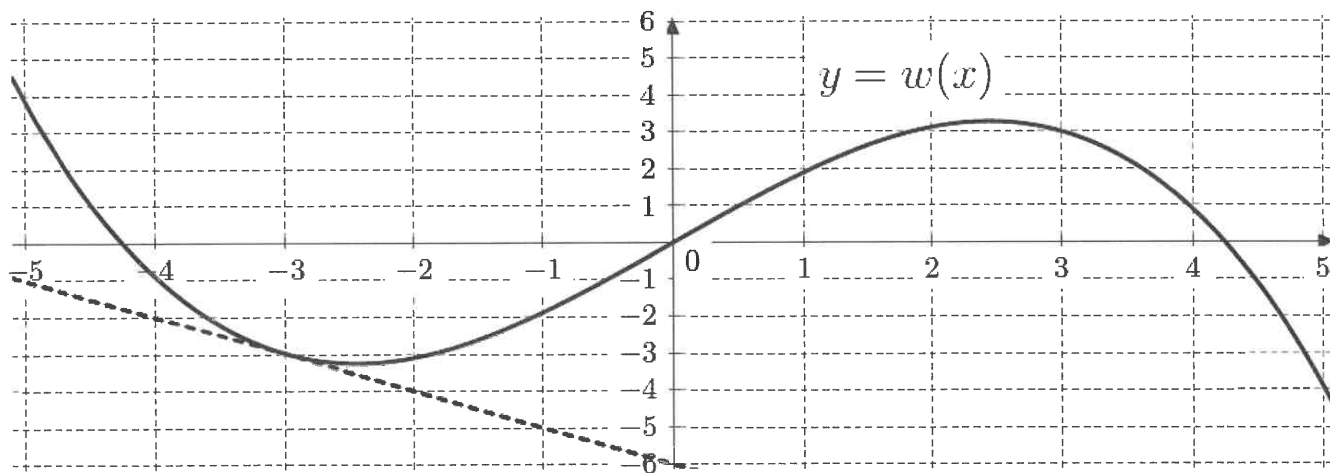
B. (3 points) $\lim_{x \rightarrow 2} \frac{x^2}{u(x) + 3w(x)}$

Problem 5. (5 points) Use the squeeze theorem to find

$$\lim_{x \rightarrow 2} g(x)$$

provided that the function $w(x)$ satisfies $3x + 1 \leq g(x) \leq x^2 + 3$ for all $x \neq 2$.

Problem 6. (4 points)



The function $y = w(x)$ is graphed above in solid bold. There is also a dotted line graphed. Find the following two values. [Answers are enough. No explanation is needed.]

A. (2 points) $w(-3) =$

B. (2 points) $w'(-3) =$

Problem 7. (10 points)

A. (5 points) $\lim_{t \rightarrow -3} \frac{t^2 - 2t - 15}{t + 3}$

B. (5 points) $\lim_{x \rightarrow 5} \frac{1 - \sqrt{x - 4}}{x - 5}$

Problem 8. (10 points) Suppose that an object is at position $s(t) = 2t^2$ feet at time t seconds.

A. (3 points) Find the average velocity of the object over a time interval from time 1 seconds to time $1 + h$ seconds.

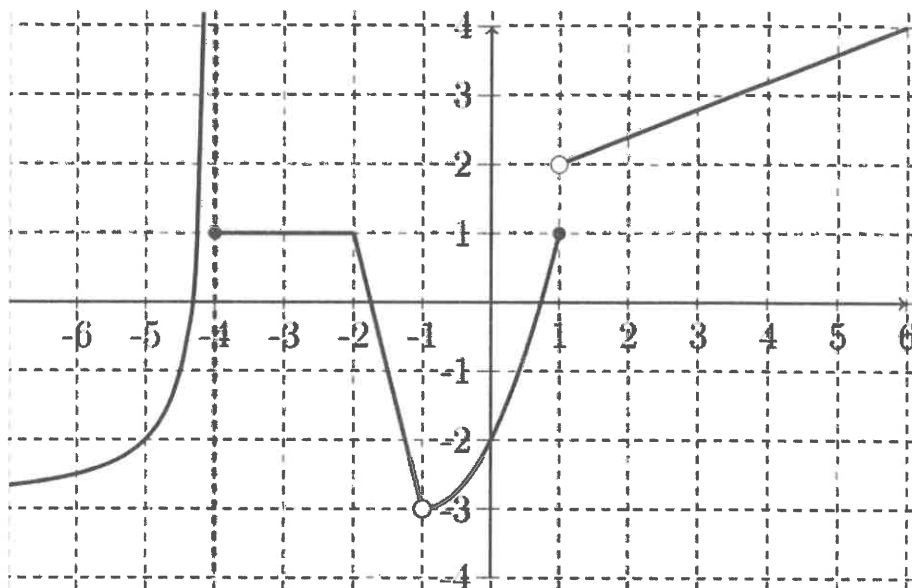
B. (7 points) Find the instantaneous velocity of the object at time 1 second by taking the limit of the average velocity in Part A as $h \rightarrow 0$.

Problem 9. (10 points) Let $v(x) = \frac{4}{x^2}$.

A. (7 points) Find $v'(2)$ by using one of the limit definitions of the derivative.

B. (3 points) Find the equation of the tangent line to $y = v(x)$ at $x = 2$.

Problem 10. (15 points)



Consider the graph $y = g(x)$ above. State the value of each of the below quantities (A - F: 2 points for each). If the quantity does not exist, write "does not exist" or "DNE". (Answers are enough. No explanation is needed.)

A. $\lim_{x \rightarrow -4^-} g(x) =$

E. $\lim_{x \rightarrow -2} g(x) =$

B. $\lim_{x \rightarrow -4^+} g(x) =$

F. $g'(-3) =$

C. $\lim_{x \rightarrow 1} g(x) =$

G. (3 points) List all discontinuities and classify them as removable, infinite or jump.

D. $\lim_{x \rightarrow -1} g(x) =$

