Math 220 Sample Midterm 2

Name: My Soln

Recitation instructor: ____________________

Recitation time: ____________________

- This is a closed-book, closed-notes exam. No calculators or electronic aids are permitted.
- Read each question carefully and show your work unless explicitly told otherwise.

Problem 1.
Find the following derivatives. You **do not need to simplify** your answers or show all steps. However, showing your work may help you earn partial credit if your answer is incorrect.

$= x^{-\frac{1}{2}}$

A. $\frac{d}{dx}\left(5x^3 - \frac{1}{\sqrt{x}} + 7\log_5(x) + e^3\right)$

$15x^2 - \frac{1}{2}x^{-\frac{3}{2}} + 7 \cdot \frac{1}{\ln 5}\,\frac{1}{x}$

B. $\frac{d}{dx}\left(20^x \cdot x^{20}\right)$

$\ln 20 \cdot 20^x \cdot x^{20} + 20^x \cdot 20x^{19}$

$= 3x^{-1} - 1$

C. $\frac{d}{dx}\arccos\left(\frac{3}{x} - 1\right)$

$-\frac{1}{\sqrt{1-\left(\frac{3}{x}-1\right)^2}} \cdot -3x^{-2}$

D. $\frac{d}{d\theta}\sec\left(\sin(\theta^2)\right)$

$$\sec(\sin(\theta^2))\tan(\sin(\theta^2)) \cdot \cos(\theta^2)\cdot 2\theta$$

E. $\frac{d}{dx}\left(\frac{e^{2x}+\ln(2x+1)}{x^6-7x}\right)$

$$\frac{(x^6-7x)\cdot\left(2e^{2x}+\frac{1}{2x+1}\cdot 2\right)-\left(e^{2x}+\ln(2x+1)\right)(6x^5-7)}{(x^6-7x)^2}$$

Problem 2. Using logarithmic differentiation, find the derivative of $f(x) = x^{7\tan(x)}$.

$$\frac{d}{dx} \quad \ln(f(x)) = 7\tan(x)\ln(x)$$

$$\frac{1}{f(x)} \cdot f'(x) = 7\left[\sec^2(x)\ln(x) + \tan(x) \cdot \frac{1}{x}\right]$$

$$\Longrightarrow f'(x) = x^{7\tan(x)}\left[7\sec^2(x)\ln(x) + \frac{7\tan(x)}{x}\right]$$

Problem 3. Using implicit differentiation, find $\frac{dy}{dx}$ if $\cos(x^2y^3) = e^x$.

$$-\sin(x^2y^3)(2xy^3 + x^2 \cdot 3y^2y') = e^x$$

$$\Rightarrow y' \cdot -\sin(x^2y^3) \cdot 3x^2y^2 = e^x + 2xy^3\sin(x^2y^3)$$

$$\Rightarrow y' = \frac{e^x + 2xy^3\sin(x^2y^3)}{-\sin(x^2y^3) \cdot 3x^2y^2}$$

Problem 4. Let $g(x) = 7x^2 + x$. Using the limit definition of the derivative, find $g'(x)$. Make sure to use limit notation correctly.

$$g'(x) = \lim_{h \to 0} \frac{g(x+h) - g(x)}{h}$$

$$= \lim_{h \to 0} \frac{7(x+h)^2 + (x+h) - [7x^2 + x]}{h}$$

$$= \lim_{h \to 0} \frac{7(\cancel{x^2} + 2xh + h^2) + (\cancel{x} + h) - \cancel{7x^2} - \cancel{x}}{h}$$

$$= \lim_{h \to 0} \frac{14xh + 7h^2 + h}{h} = \lim_{h \to 0} 14x + 7h + 1$$

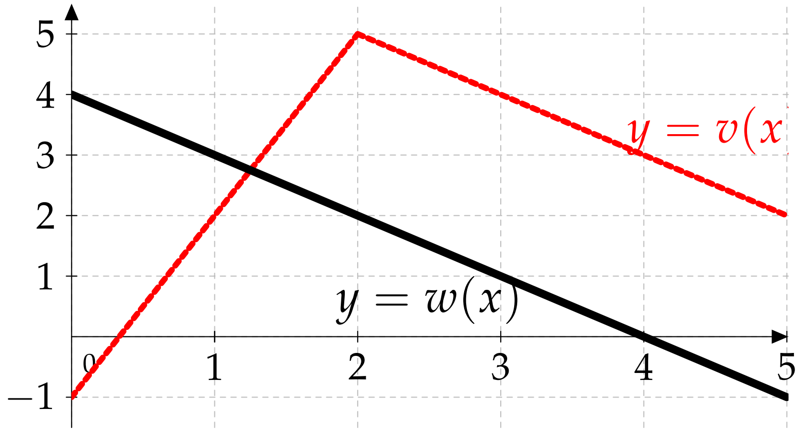
$$= 14x + 1$$

Problem 5. Suppose that a waiter brings you a cup of hot tea. Let $F(t)$ denote the temperature in degrees Fahrenheit of the tea after t minutes. Is $F'(3)$ positive or negative? Explain your answer.

$F'(3)$ is negative.

The cup's temp. should decrease over time.

Problem 6.



Suppose that $f(x) = v(x) \cdot w(x)$ and $g(x) = v(w(x))$. Find $f'(1)$ and $g'(1)$.

$f'(x) = v'(x)w(x) + v(x)w'(x)$

$f'(1) = 3(3) + 2(-1)$

$= 9 - 2 = 7$

$g'(x) = v'(w(x)) \cdot w'(x)$

$g'(1) = v'(3) \cdot (-1)$

$= (-1)(-1) = 1$

Problem 7. Find the equation of the tangent line to the curve $y = \sin(5x) + 2$ at $x = 0$.

$y' = \cos(5x) \cdot 5$

$y(0) = 0+2 = 2$

$\rightsquigarrow (0,2)$

$y'(0) = 1 \cdot 5 = 5$

$$\boxed{y - 2 = 5(x - 0)}$$

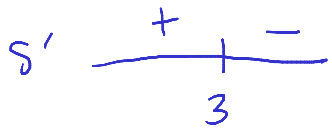
Problem 8. On an alien planet, Alice throws a softball vertically upward. For $t \geq 0$, it has height in feet given by $s(t) = 10 + 6t - t^2$, where t is in seconds.

A. Calculate $s'(t)$. When is the softball going upward/downward?

$s'(t) = 6 - 2t$

$s' = 0 \Rightarrow 6 - 2t = 0$

$\Rightarrow t = 3$



upward: $t < 3$

downward: $t > 3$

B. At what time does the softball obtain its maximum height? (**include unit with your answer.**)

At $t = 3$ sec

C. What is the acceleration $s''(t)$? (**include unit with your answer.**)

$s'' = -2 \text{ ft}/\text{sec}^2$