Math 220 Midterm 2

Name: Solution

Recitation instructor: _____

Recitation time: _____

- Exam 2 date/time: Thursday February 29, 2024 at 7:05 -8:20 pm.
- This is a closed-book, closed-notes exam. No calculators or electronic aids are permitted.
- Read each question carefully and show your work unless explicitly told otherwise.
- If you need extra room, use the blank page at the end of the exam. If you must use extra paper, make sure to write your name on it and attach it to this exam. Do not unstaple or detach pages from this exam.

Grading

1	/15	**6**	/6
2	/12	**7**	/10
3	/8	**8**	/3
4	/8	**9**	/10
5	/8	**Total**	/80

Problem 1. (*15 points*)
(5 points each) Find the following derivatives. You **do not need to simplify** your answers or show all steps. However, showing your work may help you earn partial credit if your answer is incorrect.

A. $\frac{d}{dx}\left(2x^4 + \frac{1}{x^4} - \pi^4\right)$

$= 8x^3 - 4x^{-5}$

Note:
$2(4x^3) + (-4x^{-5})$
is good as well.

B. $\frac{d}{dx}\left(7^x \cdot x^7\right)$

$= (7^x \ln 7)x^7 + 7^x(7x^6)$

C. $\frac{d}{d\theta} \arcsin(\tan(\theta) + 2\theta)$

$= \frac{1}{\sqrt{1 - (\tan\theta + 2\theta)^2}} (\sec^2\theta + 2)$

Problem 2. (*12 points*)
(6 points each) Find the following derivatives. You **do not need to simplify** your answers or show all steps. However, showing your work may help you earn partial credit if your answer is incorrect.

A. $\frac{d}{dx}\sqrt{\sqrt{x}+\ln(2x-1)}$ $= \frac{d}{dx}\left(x^{1/2}+\ln(2x-1)\right)^{1/2}$

$= \frac{1}{2}\left(x^{1/2}+\ln(2x-1)\right)^{-1/2}\left(\frac{1}{2}x^{-1/2}+\frac{2}{2x-1}\right)$

B. $\frac{d}{dx}\left(\frac{2\sec(x)-e^{x^2}}{x^{8/3}+x}\right)$

$= \frac{\left(2\sec x\tan x - e^{x^2}(2x)\right)\left(x^{8/3}+x\right) - \left(2\sec x - e^{x^2}\right)\left(\frac{8}{3}x^{5/3}+1\right)}{\left(x^{8/3}+x\right)^2}$

Problem 3. (*8 points*) Using logarithmic differentiation, find the derivative of

$$f(x) = x^{\arctan(x)+2}.$$

$$\ln f(x) = (\arctan x + 2)\ln x$$

$$\frac{f'(x)}{f(x)} = \frac{1}{1+x^2}\cdot \ln x + (\arctan x + 2)\cdot \frac{1}{x}$$

$$f'(x) = x^{\arctan x + 2}\left[\frac{\ln x}{1+x^2} + \frac{(\arctan x + 2)}{x}\right]$$

Problem 4. (*8 points*) Using implicit differentiation, find $\frac{dy}{dx}$ if $\sin(x^2y^2) = \log_5(x)$.

$$\frac{d}{dx}\sin(x^2y^2) = \frac{d}{dx}\log_5 x$$

$$\cos(x^2y^2)\frac{d}{dx}x^2y^2 = \frac{1}{x\ln 5}$$

$$\cos(x^2y^2)\left[2xy^2 + x^2(2y)\frac{dy}{dx}\right] = \frac{1}{x\ln 5}$$

$$2xy^2 + 2x^2y\frac{dy}{dx} = \frac{1}{\cos(x^2y^2)\,x\ln 5}$$

$$\frac{dy}{dx} = \frac{1}{2x^2y}\left[\frac{1}{\cos(x^2y^2)\,x\ln 5} - 2xy^2\right]$$

Problem 5. (*8 points*) Let $g(x) = 2x^2 + 5$. Using **the limit definition of the derivative**, find $g'(x)$. Make sure to use limit notation correctly.

$$g'(x) = \lim_{h \to 0} \frac{2(x+h)^2 + 5 - 2x^2 - 5}{h}$$

$$= \lim_{h \to 0} \frac{2(x^2 + 2xh + h^2) - 2x^2}{h}$$

$$= \lim_{h \to 0} \frac{4xh + h^2}{h} = \lim_{h \to 0} 4x + h$$

$$= 4x$$

Problem 6. (*6 points*) Find the equation of the tangent line to the curve $y = \cos(x) + 4$ at $x = \frac{\pi}{2}$.

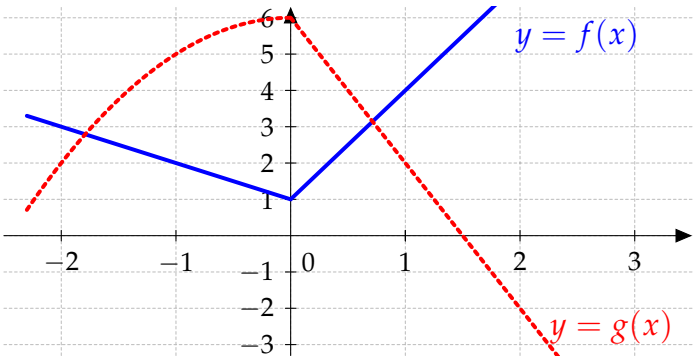
$$\frac{dy}{dx} = -\sin x$$

At $x = \frac{\pi}{2}$, $y = \cos\left(\frac{\pi}{2}\right) + 4 = 4$

At $x = \frac{\pi}{2}$, $\frac{dy}{dx} = -\sin\frac{\pi}{2} = -1$

So $y - 4 = (-1)\left(x - \frac{\pi}{2}\right)$

Problem 7. (*10 points*)



Suppose that $m(x) = f(x) \cdot g(x)$ and $p(x) = f(g(x))$. Find:

A. $m'(1)$ $= f'(1)g(1) + f(1)g'(1)$

$= 3(2) + 4(-4)$

$= 6 - 16 = -10$

B. $p'(2)$

$= f'(g(2))\, g'(2)$

$= f'(-2)\,(-4) = (-1)(-4) = 4$

Problem 8. (*3 points*) A boat hits a large rock and starts filling with water. Let $V(t)$ denote the volume of the water inside the boat t seconds after the collision. Is $V'(2)$ positive or negative? Explain your answer.

$V'(2)$ is positive because the water increases inside the boat.

Problem 9. (*10 points*) On an alien planet, Alice throws a softball vertically upward. For $t \geq 0$, it has height in feet given by $s(t) = 8 + 4t - 2t^2$, where t is in seconds.

A. (*6 points*) Calculate $s'(t)$. When is the softball going upward/downward?

$s'(t) = 4 - 4t$

upward: $s'(t) > 0 \Rightarrow 4 - 4t > 0$

$4 > 4t$

$1 > t$

downward: $s'(t) < 0 \Rightarrow \boxed{1 < t}$

B. (*2 points*) At what time does the softball obtain its maximum height? (**include unit with your answer.**)

$s'(t) = 0$

$4 - 4t = 0$

$4t = 4$

$t = 1$ sec

C. (*2 points*) What is the acceleration $s''(t)$? (**include unit with your answer.**)

$s''(t) = -4$ ft/sec^2