# Math 220 Sample Midterm 3

Name: _____

Recitation instructor: _____

Recitation time: _____

- This is a closed-book, closed-notes exam. No calculators or electronic aids are permitted.
- Read each question carefully and show your work unless explicitly told otherwise.

**Problem 1.** Let $f(x) = x^4 - 2x^2$.

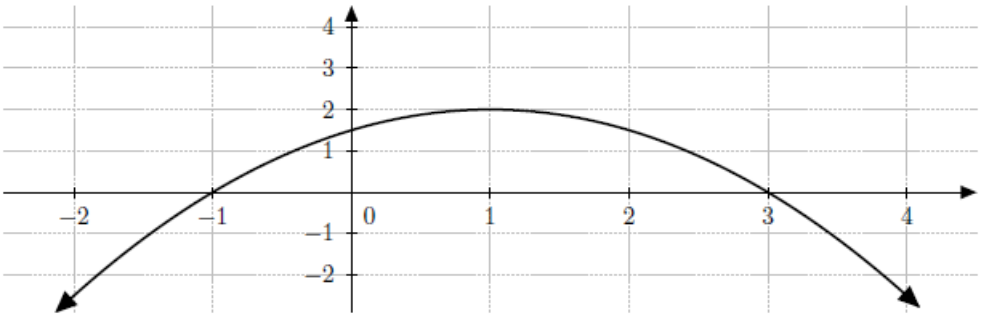
(a) ( *points*) Find all critical numbers of $f(x)$ on the interval $(-\infty, \infty)$.

(b) ( *points*) Find the absolute maximum and absolute minimum of $f(x)$ on $[-2, 2]$.

**Problem 2.** In each of the following blanks, fill in "**a local max**", "**a local min**" or "**neither**".
(Note: "**neither**" means "neither a local max nor a local min") Also, no works need to be shown.

**A.** If $h'(3) = 0$ and $h''(3) = 2$, then $h(x)$ has _____ at $x = 3$.

**B.** If $h'(-2) = 0$ and $h''(-2) = -11$, then $h(x)$ has _____ at $x = -2$.




**Above is the graph of $y = g'(x)$ to use for part C and D.**

**C.** $g(x)$ has _____ at $x = -1$.

**D.** $g(x)$ has _____ at $x = 1$.

**Problem 3.** The function $f(x)$ and its first and second derivatives are:

$$f(x) = \frac{x^2 - 9}{x^2 - 4} \qquad f'(x) = \frac{10x}{(x^2 - 4)^2} \qquad f''(x) = -\frac{10(3x^2 + 4)}{(x^2 - 4)^3}.$$

Find the information below about $f(x)$, and use it to sketch the graph of $f(x)$. When appropriate, write NONE. No work needs to be shown on this problem.

**A.** ( point) Domain of $f(x)$: _____

**B.** ( point) $y$-intercept: _____

**C.** ( point) $x$-intercept(s): _____

**D.** ( point) Horizontal asymptote(s): _____

**E.** ( point) Vertical asymptote(s): _____

**F.** ( point) Interval(s) $f(x)$ is increasing: _____

**G.** ( point) Interval(s) $f(x)$ is decreasing: _____

**H.** (0.5 point) Local maximum(s) $(x, y)$: _____

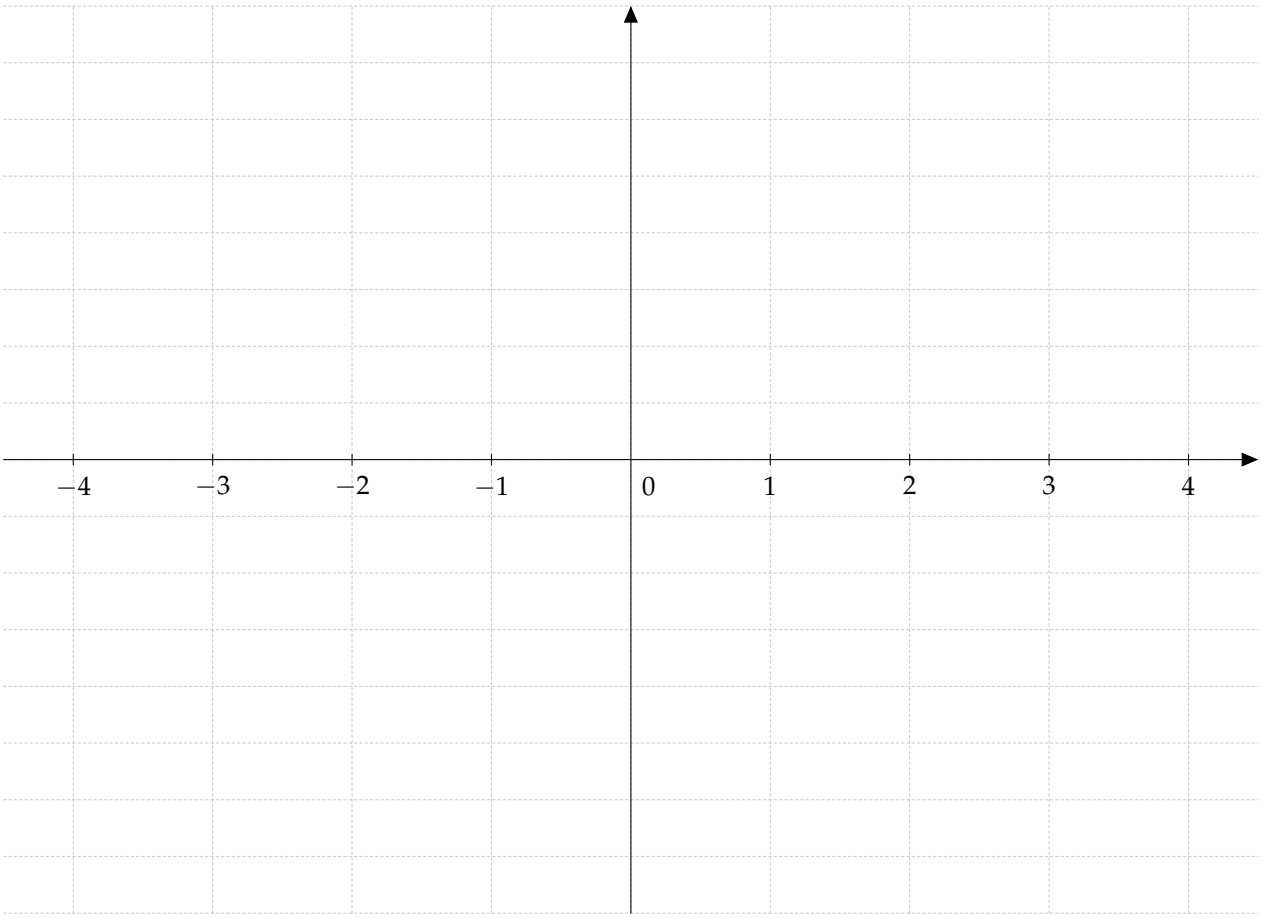
**I.** (0.5 point) Local minimum(s) $(x, y)$: _____

**J.** ( point) Interval(s) $f(x)$ is concave up: _____

**K.** ( point) Interval(s) $f(x)$ is concave down: _____

**L.** ( point) Inflection point(s) $(x, y)$: _____

**M.** Sketch $y = f(x)$ on the graph below.



**Problem 4.** Find the following limits. (Use limit notation correctly.)

**A.** ( *points*) $\lim_{x\to\infty} \dfrac{x^2-1}{e^x+2}$

**B.** ( *points*) $\lim_{x\to 1^+} (x-1)^{x-1}$

**C.** ( *points*) $\lim_{x\to-\infty} \dfrac{\sqrt{x^2+9}}{3x+2}$

**Problem 5.** Two airplanes are flying in the air at the same height: airplane A is flying east at 100 mi/h and airplane B is flying north at 200 mi/h. If they are both heading to the same airport, located 30 miles east of airplane A and 40 miles north of airplane B, at what rate is the distance between the airplanes changing? **[Include unit with your answer]**

**Problem 6.**
If 12 $ft^2$ of material is available to make a box with square base and open top, find the largest possible volume for the box. [**Include unit with your answer**]

**Problem 7.** Let $f(x) = \sin x$.

**A.** ( *points*) Find the linearization for $f(x)$ at $x = \pi$.

**B.** ( *points*) Use the linearization to approximate $f(\pi + 0.02)$.

**Problem 8.** The area $A$ of a circle with radius $r$ is given by $A = \pi r^2$. Find the differential $dA$.