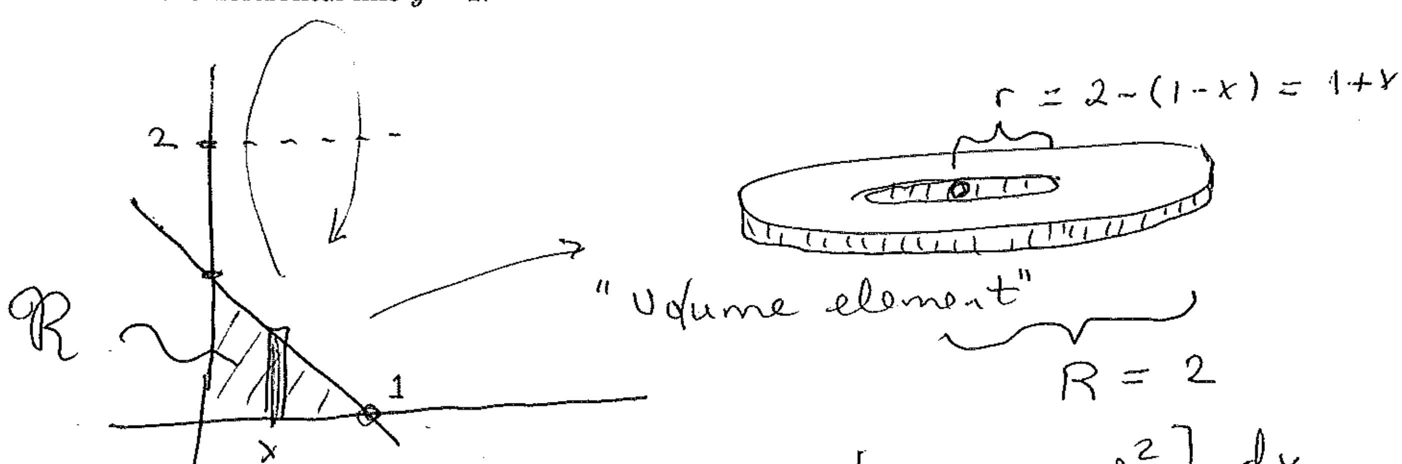
Calc 2 Exam 1 (70 possible points), Feb. 5, 2013

YOUR NAME, YOUR RECITATION INSTRUCTOR, AND HOUR OF RECITATION CLASS

For best results on this test, *draw* useful things (such as elements dV of volume, and triangles or circles needed in trigonometry problems) and label them appropriately. Clearly-labeled diagrams make it much easier to justify and assign partial credit. ALL answers must be justied, in order to receive any credit at all. EXACT NUMERICAL ANSWERS (not calculator-generated answers) ARE REQUIRED.

(14) Problem 1. Let $\mathcal{R}$ be the region bounded by the graph of $y = 1 - x$ and by the positive co-ordinate axes. Find the volume of the solid obtained by revolving $\mathcal{R}$ about the horizontal line $y = 2$.



$$dV = \pi\left(R^2 - r^2\right)dx = \pi\left[4 - (1+x)^2\right]dx$$

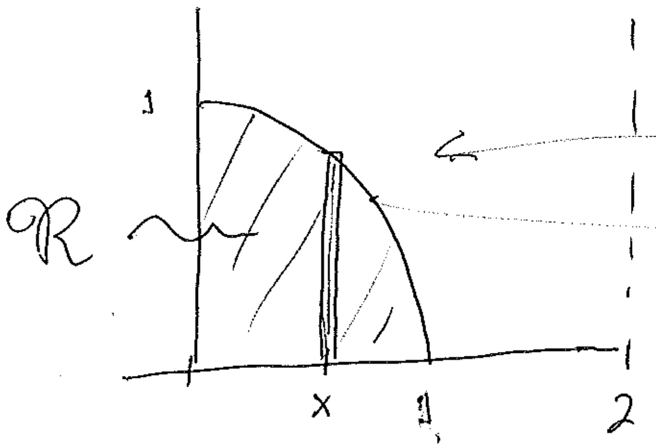
$$= \pi\left(3 - 2x - x^2\right)dx$$

$$V = \int_{x=0}^{x=1} dV = \pi\int_0^1 \left(3 - 2x - x^2\right)dx$$

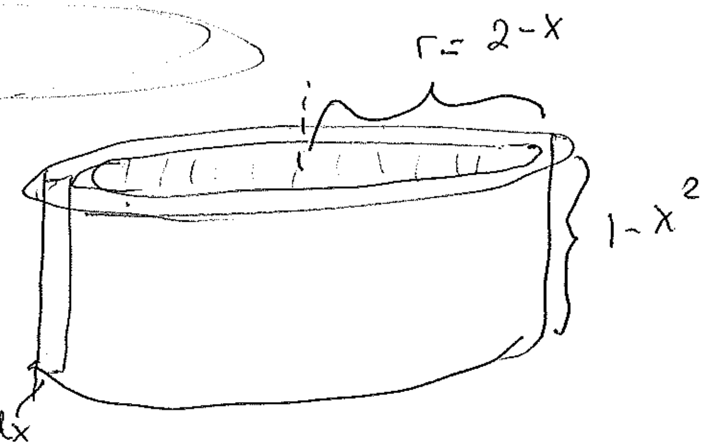
$$= \pi\left(3x - x^2 - \tfrac{1}{3}x^3\right)\Big|_0^1 = \pi\left(3 - 1 - \tfrac{1}{3}\right)$$

$$= \boxed{\frac{5\pi}{3}}$$

(14) **Problem 2.** Let $\mathcal{R}$ be the region bounded by the graph of $y = 1 - x^2$ and by the positive co-ordinate axes. Find the volume of the solid obtained by revolving $\mathcal{R}$ about the vertical line $x = 2$.



volume element →
(cylindrical shell)



$$dV = 2\pi \, r \, h \, dx = 2\pi (2-x)(1-x^2)\, dx$$

$$= 2\pi (2 - x - 2x^2 + x^3)\, dx$$

$$V = 2\pi \int_0^1 (2 - x - 2x^2 + x^3)\, dx$$

$$= 2\pi \left(2x - \frac{1}{2}x^2 - \frac{2}{3}x^3 + \frac{1}{4}x^4 \right) \Big|_0^1$$

$$= 2\pi \left(1 - \frac{1}{2} - \frac{2}{3} + \frac{1}{4} \right) = 2\pi \frac{1}{12}$$

$$= \boxed{\frac{\pi}{6}}$$

(10) **Problem 3.** Evaluate the integral $I = \int x^2\, ln(x)\, dx$. (By "parts")

$u = \ln x\ ,\quad dv = x^2\, dx$

$du = \frac{1}{x}\, dx\ ,\quad v = \frac{1}{3}x^3$

$I = uv - \int v\, du = \frac{1}{3}x^3 \ln x - \int \frac{1}{3}x^3 \frac{1}{x}\, dx$

$= \frac{1}{3}x^3 \ln x - \frac{1}{3}\int x^2\, dx$

$= \boxed{\frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 + C}$

(10) **Problem 4.** Evaluate the integral $I = \int x^2\, e^{-x}\, dx$. (By "parts")

$\left(\begin{array}{ll} u = x^2\ , & dv = e^{-x}\, dx \\ du = 2x\, dx\ , & v = -e^{-x} \end{array}\right)$

$I = uv - \int v\, du = -x^2 e^{-x} + 2\int x e^{-x}\, dx$

Next find $\int x e^{-x}\, dx$ by new parts

$\left(\begin{array}{ll} u = x\ , & dv = e^{-x}\, dx \\ du = dx\ , & v = -e^{-x} \end{array}\right) \to \int x e^{-x}\, dx = -x e^{-x} + \int e^{-x}\, dx$

$= -x e^{-x} - e^{-x} + C$

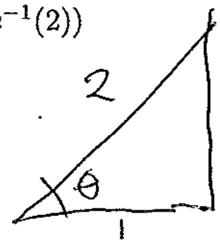
$\boxed{I = -x^2 e^{-x} + 2\left(-x e^{-x} - e^{-x}\right) + C}$

("same" as 2C since C is an arbitrary constant)

(15) Problem 5. Find the exact numerical values of each of the following expressions. [Hint for part (c): Use an appropriate trigonometric identity.]

(a) $tan(sec^{-1}(2))$

$\theta = \sec^{-1} 2$

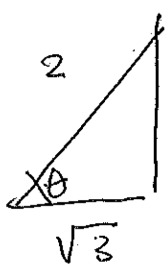


$\tan\theta = \boxed{\sqrt{3}}$

(b) $sin^{-1}(sin(\frac{1}{e}))$ $= \boxed{\frac{1}{e}}$ (since $\sin^{-1}(\sin x) = x$ for all x between -1 and 1)

(c) $sin(2\ sin^{-1}(\frac{1}{2}))$

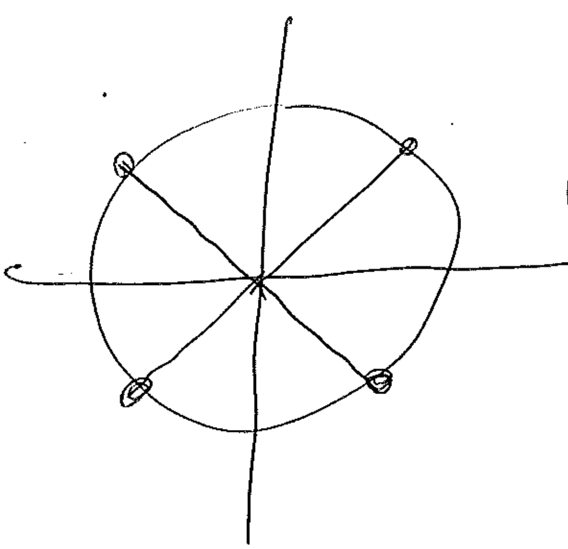
$\theta = \sin^{-1}(1/2)$, $\sin 2\theta = 2\sin\theta\cos\theta$



$\sin(2\theta) = 2 \cdot \frac{1}{2} \frac{\sqrt{3}}{2} = \boxed{\frac{\sqrt{3}}{2}}$

(7) Problem 5. Find all of the solutions, in the interval $[0, 2\pi]$, to the equation

$$|sin\ t| = |cos\ t|$$



$\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$