Math 221 Calculus 2
Professor John Maginnis

Midterm Exam 1
September 23, 2014

Your name: Solutions

Rec. Instr.: ____________ Rec. Time: ____________

~~Show all your work in the space provided under each question.~~ ~~Please write neatly~~ and present your answers in an organized way. You may use your one sheet of notes, but no books or calculators. This exam is worth 60 points. The chart below indicates how many points each problem is worth.

| Problem | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Points | /8 | /8 | /8 | /8 |
| Problem | 5 | 6 | 7 | 8 |
| Points | /6 | /10 | /4 | /8 |

1. Evaluate the integral.

$$\int \sin^3(x)\cos^4(x)\,dx = \int \sin(x)\cdot\sin^2(x)\cdot\cos^4(x)\,dx$$

$$= \int \sin(x)\cdot(1-\cos^2(x))\cdot\cos^4(x)$$

Use the substitution $u=\cos(x)$, $du=-\sin(x)\,dx$

$$\int -(1-u^2)u^4\,du = \int u^6-u^4\,du = \frac{1}{7}u^7-\frac{1}{5}u^5+C$$

$$= \boxed{\frac{1}{7}\cos^7(x)-\frac{1}{5}\cos^5(x)+C}$$

2. Evaluate the integral.

$$\int x^3\cos(x^2)\,dx$$

Substitution $w=x^2$, $dw=2x\,dx$
$\frac{1}{2}dw = x\,dx$

$$\int x\cdot x^2\cdot\cos(x^2)\,dx = \frac{1}{2}\int w\cdot\cos(w)\,dw$$

Integration by Parts: $u=w$, $dv=\frac{1}{2}\cos(w)\,dw$
$du=dw$, $v=\frac{1}{2}\sin(w)$

$$\int x^3\cos(x^2)\,dx = \frac{1}{2}w\cdot\sin(w)-\int\frac{1}{2}\sin(w)\,dw$$

$$= \frac{1}{2}w\cdot\sin(w)+\frac{1}{2}\cos(w)+C$$

$$= \boxed{\frac{1}{2}x^2\sin(x^2)+\frac{1}{2}\cos(x^2)+C}$$

3. Use the substitution $u = x^2 + 1$ to evaluate the integral.

$$\int x\sqrt{x^2+1}\,dx$$

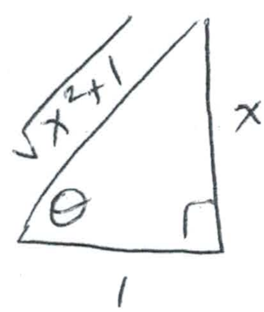
$$u = x^2 + 1$$
$$du = 2x\,dx$$
$$\frac{1}{2}\,du = x\,dx$$

$$\frac{1}{2}\int \sqrt{u}\,du = \frac{1}{2}\left(\frac{u^{3/2}}{3/2}\right) + C = \frac{1}{3}u^{3/2} + C$$

$$= \boxed{\frac{1}{3}(x^2+1)^{3/2} + C}$$

4. Use the trigonometric substitution $x = \tan(\theta)$ to evaluate the integral.

$$\int x\sqrt{x^2+1}\,dx$$




$$x = \tan(\theta)$$
$$dx = \sec^2(\theta)\,d\theta$$
$$\sqrt{x^2+1} = \sec(\theta)$$

$$\int \tan(\theta)\cdot\sec(\theta)\cdot\sec^2(\theta)\,d\theta$$

Substitute $u = \sec(\theta)$, $du = \sec\theta\tan\theta\,d\theta$

$$\int u^2\,du = \frac{1}{3}u^3 + C = \frac{1}{3}\sec^3\theta + C = \boxed{\frac{1}{3}(x^2+1)^{3/2} + C}$$

5. Evaluate the limit.

$$\lim_{x\to\infty} \frac{\ln(x)}{\sqrt{x}} = \frac{\infty}{\infty}$$, so can use <u>L'Hôpital's Rule</u>

$$\lim_{x\to\infty} \frac{\frac{1}{x}}{\frac{1}{2\sqrt{x}}} = \lim_{x\to\infty} \frac{2\sqrt{x}}{x} = \lim_{x\to\infty} \frac{2}{\sqrt{x}} = \frac{2}{\infty} = \boxed{0}$$

6. Evaluate the improper integral.

$$\int_1^\infty \frac{dx}{2x^2+3x-2} = \lim_{R\to\infty} \int_1^R \frac{dx}{2x^2+3x-2}$$

$2x^2+3x-2 = (2x-1)(x+2)$; note $x = \frac{1}{2}$ or $-2$ not in interval.

$$\frac{1}{2x^2+3x-2} = \frac{A}{2x-1} + \frac{B}{x+2}, \quad 1 = A(x+2) + B(2x-1)$$

Either $\begin{cases} A+2B=0 \\ 2A-B=1 \end{cases}$ or if $x=-2$, $1=-5B$, $B=-1/5$

or if $x = 1/2$, $1 = A\left(\frac{5}{2}\right)$, $A = 2/5$

$$\text{So } \int \frac{dx}{2x^2+3x-2} = \int \frac{2}{5}\cdot\frac{1}{2x-1} - \frac{1}{5}\cdot\frac{1}{x+2}\,dx =$$

$$\frac{1}{5}\ln|2x-1| - \frac{1}{5}\ln|x+2| + C = \frac{1}{5}\ln\left|\frac{2x-1}{x+2}\right| + C.$$

$$\text{Thus } \int_1^\infty \frac{dx}{2x^2+3x-2} = \lim_{R\to\infty}\left[\frac{1}{5}\ln\left|\frac{2x-1}{x+2}\right|\right]_1^R =$$

$$\lim_{R\to\infty}\left(\frac{1}{5}\ln\left(\frac{2R-1}{R+2}\right) - \frac{1}{5}\ln\left(\frac{1}{3}\right)\right) = \boxed{\frac{1}{5}\ln(2) - \frac{1}{5}\ln\left(\frac{1}{3}\right)} = \frac{\ln(6)}{5}$$

7. Give the form for the partial fraction decomposition, but do not solve for the coefficients. (Example: $\frac{1}{x^2-1} = \frac{A}{x-1} + \frac{B}{x+1}$.)

$$\frac{1}{x^3(x^2+1)^2} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{Dx+E}{x^2+1} + \frac{Fx+G}{(x^2+1)^2}$$ is the best answer. The next best answer would be

$$\frac{Ax^2+Bx+C}{x^3} + \frac{Dx^3+Ex^2+F'x+G'}{(x^2+1)^2}$$

8. Evaluate the integral.

$$\int \frac{\ln(x)}{x^4}\,dx$$

Integration by Parts with

$u = \ln(x)$ and $dv = x^{-4}\,dx$

$du = \frac{1}{x}\,dx$ $\quad v = -\frac{1}{3}x^{-3}$

$$\int \frac{\ln(x)}{x^4}\,dx = -\frac{1}{3}\frac{\ln(x)}{x^3} - \int -\frac{1}{3}x^{-4}\,dx$$

$$= \frac{-\ln(x)}{3x^3} + \frac{1}{3}\left(-\frac{1}{3}x^{-3}\right) + C$$

$$= \boxed{\frac{-\ln(x)}{3x^3} - \frac{1}{9x^3} + C}$$