Math 221 Spring 2014 — Midterm Exam 1
Professor Reznikoff — February 6, 2014

Your name: ______________________________

Rec. Instr.: ______________________________

Rec. Time: ______________________________

Instructions:

Show all your work in the space provided under each question. Please write neatly and present your answers in an organized way. You should leave values such as π or $\sqrt{3}$ or $\sqrt{2}$ as part of your answers (do not approximate them with decimals).

Note that some equations have been provided for you on the **last** page—rip it off for reference if you like. Use of other notes or calculators is not permitted.

Problem	1	2	3	4
Points	/5	/5	/5	/5
Problem	5	6	7	Total
Points	/5	/5	/5	/35

1. $\int \sin\theta \sqrt{\cos\theta}\,\mathrm{d}\theta$

2. $\displaystyle\int_0^{\pi/4} \sec^3 x \tan^3 x \, \mathrm{d}x$

3. $\int x^2 e^{2x}\,\mathrm{d}x$

4. $\int \cosh^4 x \sinh^3 x \, \mathrm{d}x$

5. $\displaystyle\int \frac{\mathrm{d}x}{(x^2+9)^{3/2}}$

6. Use the definition of the hyperbolic cosine to show that

$$\cosh^2 x = \frac{1}{2}\left(\cosh(2x) + 1\right).$$

7. Consider the integral $\displaystyle\int \frac{\mathrm{d}x}{x\sqrt{x^2-1}}$

(a) Evaluate the integral using trigonometric substitution.

(b) Make a hyperbolic trigonometric substitution to the integral and simplify (but do not actually integrate).

(c) Use your answers to (a) and (b) to conclude that

$$\int \operatorname{sech}\theta \,\mathrm{d}\theta = \sec^{-1}(\cosh\theta) + C.$$

$$
\begin{aligned}
\int \sec x \,\mathrm{d}x &= \ln|\sec x + \tan x| + C \\
\int \csc x \,\mathrm{d}x &= -\ln|\csc x + \cot x| + C
\end{aligned}
$$

$$
\begin{aligned}
\int \cos^n x &= \frac{\cos^{n-1} x \sin x}{n} + \frac{n-1}{n} \int \cos^{n-2} x \,\mathrm{d}x \\
\int \sec^m x &= \frac{\tan x \sec^{m-2} x}{m-1} + \frac{m-2}{m-1} \int \sec^{m-2} x \,\mathrm{d}x
\end{aligned}
$$

$$
\begin{aligned}
\cosh^2 x - \sinh^2 x &= 1 \\
\tanh^2 x + \operatorname{sech}^2 x &= 1
\end{aligned}
$$

$$
\frac{\mathrm{d}}{\mathrm{d}x} \tanh x = \operatorname{sech}^2 x \qquad \frac{\mathrm{d}}{\mathrm{d}x} \operatorname{sech} x = -\operatorname{sech} x \tanh x
$$

$$
\frac{\mathrm{d}}{\mathrm{d}x} \coth x = -\operatorname{csch}^2 x \qquad \frac{\mathrm{d}}{\mathrm{d}x} \operatorname{csch} x = -\operatorname{csch} x \coth x
$$