

Math 221 Spring 2014
Professor Reznikoff

Midterm Exam 3
April 8, 2014

Your name: _____

Rec. Instr.: _____

Rec. Time: _____

Instructions: Show all your work in the space provided under each question. Please write neatly and present your answers in an organized way. Leave values such as π or $\sqrt{3}$ or $\sqrt{2}$ as part of your answers.

Problem	1	2	3	4
Points	/12	/12	/6	/10
Problem	5(a,b)	5(c)	6	Total
Points	/12	/6	/12	/70

Note: all midterms will have the same weight.

1. Decide whether the given **sequence** converges or diverges. If it converges, give the limit, and if it diverges, explain briefly why.

(a) $(a_n)_{n=1}^{\infty}$, where $a_n = \frac{n^2 + n}{4n^2 - n + 1}$

(b) $(a_n)_{n=1}^{\infty}$, where $a_n = \frac{1 + (-1)^n}{2}$.

(c) $(a_n)_{n=1}^{\infty}$, where $a_n = \frac{\sin n}{e^n}$

2. Decide whether the series converges or diverges. If it converges, **evaluate it**.

(a) $\sum_{n=0}^{\infty} \frac{3^{n+1}}{2^{3n}}$

(2 continued) Decide whether the series converges or diverges. If it converges, **evaluate it**.

(b)
$$\sum_{n=0}^{\infty} \frac{7^n}{5^{n+1}}$$

(c)
$$\sum_{n=1}^{\infty} \left(\frac{1}{\sqrt{n+1}} - \frac{1}{\sqrt{n+2}} \right)$$

3. Find the center of mass of the region of the plane bounded by the curve $y = 1 - x^2$ and the x -axis.

4. (a) Find the Taylor polynomial T_2 centered at $a = 1$ for the function $f(x) = \ln x$.
- (b) Use your answer to part (a) to approximate $\ln \frac{3}{2}$.
- (c) Find an upper bound for the error in the approximation.

5. Determine whether the series converges. Justify your answer.

(a)
$$\sum_{n=1}^{\infty} \frac{n^2 3^n}{n!}$$

(b)
$$\sum_{n=1}^{\infty} \frac{2n^3 + n - 1}{n^5 + n^2}$$

(c) $\sum_{n=1}^{\infty} \left(1 - \cos \frac{1}{n^2}\right)$

6. Determine whether the series converges absolutely, converges conditionally, or diverges.

(a)
$$\sum_{n=0}^{\infty} (-1)^n \left(\frac{n^2}{3n^2 + 4} \right)^n$$

(b)
$$\sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt{n}}$$