Math 221 Final Exam

Kansas State University   Spring 2014

Last name ....

First name ....

WID ....

Recitation Section ....

Email ....

1. (8 pts) Integrate: $\displaystyle\int \frac{4+x}{x(x-2)^2}\,\mathrm{d}x$

2. (8 pts) Evaluate: $\displaystyle\int_1^{e^2} \ln t \, \mathrm{d}t$

3. (8 pts) Integrate: $\displaystyle\int \frac{\sqrt{x^2 - 25}}{x}\,\mathrm{d}x$

4. (10 pts)

a) Find the degree two Maclaurin polynomial T_2 for the function $f(x) = \sqrt{x+4}$.

b) Find a bound for the error when T_2 is used to approximate $\sqrt{5}$.

5. (8 pts)

a) Evaluate the limit. $\lim_{x \to \infty} (1 + 3x)^{1/x}$

b) Determine whether the series converges: $\sum_{n=1}^{\infty} (1 + 3n)^{1/n}$

6. (10 pts) If the series converges, evaluate it (determine what it equals). If it diverges, explain why.

a) $\displaystyle\sum_{n=0}^{\infty}(-1)^n\frac{\pi^{2n}}{(2n)!}$

b) $\displaystyle\sum_{n=0}^{\infty}(-1)^n\frac{5^n}{2^{3n}}$

c) $\displaystyle\sum_{n=0}^{\infty}(-1)^n$

7. (10 pts) Determine the radius and interval of convergence of the power series: $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{2^n(n+1)}(x-5)^n.$

8. (10 pts)

a) Graph the polar curve $r = 4\sin\theta, \qquad 0 \le \theta \le 2\pi$.

b) Find the area in the first quadrant bounded by the curve, the y-axis, and the line $\theta = \pi/4$.

9. (10 pts) Find the Maclaurin series for the function

$$f(x) = \int_0^x e^{-t^2}\,\mathrm{d}t.$$

10. (10 pts) Determine whether the series converges and justify your answer.

a) $$\sum_{n=1}^{\infty} \frac{2n^3 - n}{n^7 - 1}$$

b) $$\sum_{n=1}^{\infty} \frac{n!}{(2n)!}$$

11. (10 pts)

a) Determine whether the integral converges or diverges: $\displaystyle\int_2^\infty \frac{\mathrm{d}x}{x\ln x}$.

b) Determine whether the series converges absolutely, converges conditionally, or diverges: $\displaystyle\sum_{n=2}^\infty (-1)^n \frac{1}{n\ln n}$.

12. (12 pts) Consider the parameterized curve $x = t^2$, $y = t^3$, $0 \leq t \leq 2$.

a) Find the length of the curve.

b) Write an integral to express the surface area of the surface obtained by rotating the curve about the **y**-axis.

c) Find an equation for the tangent line at $t = 1$.

d) Find the value of the second derivative $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$ at $t = 1$.