NAME Solutions Rec. Instructor: ____________

Signature ____________ Rec. Time ____________

CALCULUS II - EXAM 1

September 22, 2015

Show all work for full credit. No books, notes or calculators are permitted. The point value of each problem is given in the left-hand margin. You have 65 minutes.

Problem	Points	Points Possible	Problem	Points	Points Possible
1a		12	5a		8
1b		12	5b		8
2		12	5c		8
3		12	6a		8
4		12	6b		8
			Total Score		100

$\frac{d}{dx}\tan x = \sec^2 x$ $\quad\quad$ $\frac{d}{dx}\sec x = \sec x \tan x$

$\frac{d}{dx}\sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}}$ $\quad\quad$ $\frac{d}{dx}\tan^{-1}(x) = \frac{1}{1+x^2}$ $\quad\quad$ $\frac{d}{dx}\sec^{-1}(x) = \frac{1}{|x|\sqrt{x^2-1}}$

$$\int \tan x\, dx = -\ln|\cos x| + C \quad\quad \int \sec x\, dx = \ln|\sec x + \tan x| + C$$

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C \quad\quad \int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \sin^n x\, dx = -\frac{\sin^{n-1} x \cos x}{n} + \frac{n-1}{n}\int \sin^{n-2} x\, dx,$$

$$\int \tan^n x\, dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x\, dx,$$

$$\int \sec^n x\, dx = \frac{\sec^{n-2} x \tan x}{n-1} + \frac{n-2}{n-1}\int \sec^{n-2} x\, dx$$

1. Evaluate the following integrals.

(12) a) $\int x^2 \ln(x)\, dx$

$u = \ln(x)$

$dv = x^2 dx$ $\quad du = \frac{1}{x} dx$

$v = x^3/3$

$$= uv - \int v\, du = \ln(x) \cdot \frac{x^3}{3} - \int \frac{x^3}{3} \cdot \frac{1}{x}\, dx$$

$$= \frac{1}{3} x^3 \ln(x) - \frac{1}{3} \int x^2 dx$$

$$= \frac{1}{3} x^3 \ln(x) - \frac{1}{3} \frac{x^3}{3} + C$$

$$= \frac{1}{3} x^3 \ln(x) - \frac{1}{9} x^3 + C$$

(12) b) $\int \sin^4(x)\, dx$ *Use reduction formula on cover with $n=4$*

$$= -\frac{\sin^3 x \cos x}{4} + \frac{3}{4} \int \sin^2 x\, dx$$

Again use reduction formula with $n=2$

$$= -\frac{1}{4} \sin^3 x \cos x + \frac{3}{4} \left[-\frac{\sin x \cos x}{2} + \frac{1}{2} \int 1\, dx \right]$$

$$= -\frac{1}{4} \sin^3 x \cos x - \frac{3}{8} \sin x \cos x + \frac{3}{8} x + C$$

(12) 2. Evaluate the following integral using an appropriate trig substitution.

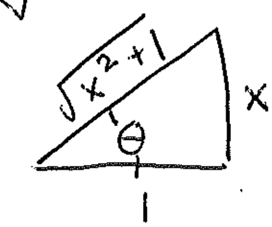
$$\int \frac{dx}{\sqrt{1+x^2}}$$

$x = \tan\theta$

$dx = \sec^2\theta \, d\theta$

$$= \int \frac{\sec^2\theta \, d\theta}{\sqrt{1+\tan^2\theta}} = \int \frac{\sec^2\theta}{\sec\theta} \, d\theta = \int \sec\theta \, d\theta$$

$$= \ln|\sec\theta + \tan\theta| + C$$



$$= \ln\left|\sqrt{x^2+1} + x\right| + C$$

(12) 3. Evaluate the following integral using an appropriate substitution.

$$\int \frac{e^x}{(1+e^x)^3} \, dx$$

$u = 1 + e^x$

$du = e^x \, dx$

$$= \int \frac{du}{u^3} = \int u^{-3} \, du = \frac{u^{-2}}{-2} = -\frac{1}{2u^2}$$

$$= -\frac{1}{2} \frac{1}{(1+e^x)^2} + C$$

(12) 4. Evaluate the integral $\int \frac{x+5}{x^3+x}\,dx$

$$\frac{x+5}{x^3+x} = \frac{x+5}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$$

$$x+5 = A(x^2+1) + (Bx+C)x$$

$$0x^2+x+5 = (A+B)x^2 + Cx + A$$

Equating constant term: $A = 5$

" x term: $C = 1$

" x^2 term: $A+B=0$, $B=-5$

$$\int \frac{x+5}{x^3+x}\,dx = \int \frac{5}{x}\,dx + \int \frac{-5x}{x^2+1}\,dx + \int \frac{1}{x^2+1}\,dx$$

$u = x^2+1$

$du = 2x\,dx$

$$= 5\ln|x| - 5\int \frac{du/2}{u} + \tan^{-1}(x) + C$$

$$= 5\ln|x| - \frac{5}{2}\ln|x^2+1| + \tan^{-1}(x) + C$$

5. Evaluate the following limits or indicate that they diverge.

(8) a) $\lim_{x\to 0} \frac{e^{2x}-2x-1}{x^2}$ $\frac{0}{0}$ type, so we can apply L'Hopital

$$= \lim_{x\to 0} \frac{2e^{2x}-2}{2x} = \lim_{x\to 0} \frac{4e^{2x}}{2} = \frac{4e^0}{2} = 2$$

$\frac{0}{0}$ type L'Hopital

(8) b) $\lim_{x\to\infty} \frac{e^{2x}+x}{e^{3x}+5}$ $\frac{\infty}{\infty}$ type, so we can apply L'Hopital

$$= \lim_{x\to\infty} \frac{2e^{2x}+1}{3e^{3x}} = \lim_{x\to\infty} \frac{4e^{2x}}{9e^{3x}}$$

$\frac{\infty}{\infty}$ type

$$= \lim_{x\to\infty} \frac{4}{9e^{x}} = 0,$$

since $e^x \to \infty$

(8) c) $\lim_{x\to\infty} \left(\frac{x+1}{x}\right)^{2x} = L$

$$\ln(L) = \lim_{x\to\infty} \ln\left(1+\frac{1}{x}\right)^{2x} = \lim_{x\to\infty} 2x \ln\left(1+\frac{1}{x}\right)$$

$$= \lim_{x\to\infty} \frac{2\ln\left(1+\frac{1}{x}\right)}{1/x} = \lim_{x\to\infty} \frac{\frac{2}{\left(1+\frac{1}{x}\right)}\cancel{\left(-\frac{1}{x^2}\right)}}{\cancel{\left(-\frac{1}{x^2}\right)}}$$

L'Hopital

$$= \lim_{x\to\infty} \frac{2}{1+1/x} = \frac{2}{1} = 2$$

Thus, $L = e^2$

6. Evaluate the improper integrals or show that they diverge. Make careful use of limit notation.

(8) a) $\int_2^5 \frac{dx}{\sqrt{x-2}} = \lim_{t \to 2^+} \int_t^5 (x-2)^{-\frac{1}{2}}\, dx$

$= \lim_{t \to 2^+} 2(x-2)^{1/2} \Big|_t^5 = \lim_{t \to 2^+} 2 \cdot 3^{1/2} - 2(t-2)^{1/2}$

$= 2\sqrt{3} - 2 \cdot 0$

$= 2\sqrt{3}$

(8) b) $\int_3^\infty \frac{1}{(x-1)^3}\, dx = \lim_{t \to \infty} \int_3^t (x-1)^{-3}\, dx$

$= \lim_{t \to \infty} \frac{(x-1)^{-2}}{-2} \Big|_3^t = \lim_{t \to \infty} -\frac{1}{2} \frac{1}{(x-1)^2} \Big|_3^t$

$= \lim_{t \to \infty} \left(-\frac{1}{2} \frac{1}{(t-1)^2} + \frac{1}{2} \frac{1}{2^2} \right)$

$= 0 + \frac{1}{8} = \frac{1}{8}$