NAME _____ Rec. Instructor: _____

Signature _____ Rec. Time _____

CALCULUS II - EXAM 2
October 20, 2015

Show all work for full credit. No books, notes or calculators are permitted. The point value of each problem is given in the left-hand margin. You have 65 minutes.

Problem	Points	Points Possible	Problem	Points	Points Possible
1		10	7a		6
2		10	7b		6
3		12	8		4
4		12	9a		5
5		12	9b		5
6		12	9c		6
			Total Score		100

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}(\frac{x}{a}) + C \qquad \int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}(\frac{x}{a}) + C$$

$$\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}(\frac{|x|}{a}) + C$$

$$\int \sqrt{a^2-u^2}\, du = \frac{1}{2}\left(u\sqrt{a^2-u^2} + a^2\sin^{-1}\frac{u}{a}\right) + C\,,$$

$$\int \sqrt{u^2\pm a^2}\, du = \frac{1}{2}\left(u\sqrt{u^2\pm a^2} \pm a^2\ln|u+\sqrt{u^2\pm a^2}|\right) + C$$

Units of force: pounds, newtons; Gravitational acceleration: $g = 9.8m/sec^2$
Work = Force $\times$ Distance; Units of work: ft-lbs, newton-meters = joules;
Hooke's Law for springs: $F = kx$, where x is the distance stretched from rest position.

Moments: For the region between $y = f(x)$ and $y = g(x)$, with $a \le x \le b$,
$M_x = \frac{1}{2}\int_a^b f(x)^2 - g(x)^2\, dx$, $M_y = \int_a^b x(f(x)-g(x))\, dx$.

Taylor Remainder: $|R_n(x)| \le \frac{K}{(n+1)!}|x-a|^{n+1}$, with $K = \max_{a\le c\le x}|f^{(n+1)}(c)|$.

(10) 1. Calculate the length of the curve $y = x^2$, $0 \leq x \leq 1$. (Make use of an appropriate integral on the cover page if necessary.)

(10) 2. Calculate the surface area of the surface obtained by rotating the curve $y = x^3$, $0 \leq x \leq 1$, about the x-axis.

(12) 3. Calculate the amount of work required to pull a spring 1 foot beyond its rest length, if the force required to do so is 6 pounds.

(12) 4. Find the centroid of the region bounded by the semicircle $y = \sqrt{4 - x^2}$, $-2 \leq x \leq 2$ and the x-axis. (You may use symmetry and the area formula for a semicircle.)

(12) 5. Find the second degree Taylor polynomial for the function $f(x) = xe^x$ about $x = 1$.

(12) 6. Solve the differential equation with initial condition,

$$\frac{dy}{dx} = \frac{\ln x}{xy}, \qquad y(1) = 2, \quad (x > 0).$$

Your final answer should be in the form $y = f(x)$ for some function $f(x)$.

7. Find the limit of the following sequences, or explain why they diverge.

(6) a) $\lim_{n\to\infty} \frac{n^2}{e^n} =$

(6) b) $\lim_{n\to\infty} n\tan(\pi/n)$

(4) 8. Find a formula for the general term a_n of the series $2 - \frac{2^2}{2!} + \frac{2^3}{3!} - \frac{2^4}{4!} + \cdots$
(You do **not** need to calculate any sum.)

9. Evaluate the following series, or state that it diverges and explain why it diverges.

(5) a) $\frac{3}{2} + \frac{1}{2} + \frac{1}{6} + \frac{1}{18} + \frac{1}{54} + \cdots$

(5) b) $\sum_{k=1}^{\infty} \frac{2k-1}{7k+5}$

(6) c) $\sum_{n=2}^{\infty} \frac{2}{n^2-1}$