MATH 221, Calculus II
Professor Hrant Hakobyan
7:05-8:20 PM

Midterm Exam 2
March 3, 2015

Your Name: _____

Recitation Instructor: _____

Recitation Time:_____

Show all your work in the space provided under each problem. Please write and present your answer in an organized way. You may use your one sheet of notes but no books or calculators. The exam is worth 60 points. The chart below indicates how many points each problem is worth. You may use the last page for your calculations.

Problem	1	2	3	4	5	6	7	**Total:**
Points								
Out of	6	6	9	10	9	10	10	60

(1) Compute the limit, or indicate why it does not exist

(a) $\lim_{x\to 0} \dfrac{x^2}{1-\cos(2x)}$

(b) $\lim_{x\to\infty} \dfrac{\ln(x^5+1)}{x}$

(c) $\lim_{x\to 0} (\cos x)^{4/x^2}$. Hint: You may need the following limit $\lim_{x\to 0} \dfrac{\sin x}{x} = 1$.

(2) Compute the integral

$$\int \frac{x^2 + 11x}{(x-1)(x+1)^2}\, dx$$

(3) (*a*) Compute the integral or indicate why it diverges

$$\int_0^1 \ln(1-x)\,dx$$

(*b*) Use integration by parts and L'Hopital's rule to find the improper integral.

$$\int_2^\infty \frac{\ln x}{x^2} dx$$

(4) Calculate the arclength of the graph of

$$y = \frac{3}{8}x^{4/3} - \frac{3}{4}x^{2/3}$$

over $[0, 1]$.

(5) Calculate the surface area of a surface obtained by rotating the part of the graph of $f(x) = e^x$ over $[0, 1]$ around the x-axis.

Hint: You may need the following formula

$$\int \sqrt{1+u^2}\, du = \frac{u}{2}\sqrt{1+u^2} + \frac{1}{2}\ln(u + \sqrt{1+u^2})$$

(6) Find the centroid of the region lying between the graphs of the functions $y = \sqrt{x}$, and $y = x^2$ over the interval $[0, 1]$.

Hint: For finding y_{CM} you may use the formula $M_x = \dfrac{\rho}{2} \displaystyle\int_a^b x(f_1(x)^2 - f_2(x)^2)dx$.

(7) Calculate the fluid force on the Three Gorges Dam (the largest dam in the world) in China. Assume, that the side of the dam facing the river is of the form of a vertical wall, which has a shape of a trapezoid with base of 2000 meters, the top edge of 3000 meters and height of 200 meters. You may assume that the density of the water is ρ kg/m^3 and the gravitational acceleration is g m/sec^2 and leave your answer in terms of ρ and g.

Hint: The length of the cross-section at depth y is given by $f(y) = 3000 - m \cdot y$ for some constant m, which may be found from the provided data.

...