MATH 221, Calculus II
Final Examination

May 13, 2015
6:20-8:10 PM

Your Name: _____

Recitation Instructor: _____

Recitation Time: _____

Show all your work in the space provided under each problem. You may use back side of the page for your calculations.

Please write and present your answer in an organized way. You may use your one sheet of notes but no books or calculators.

When calculating integrals, write which method of integration you are using. For each test of convergence that you use, either give the name of the test, or briefly describe what the test says.

The exam is worth 120 points. The chart below indicates how many points each problem is worth.

Problem	1	2	3	4	
Points/Out of	/20	/20	/16	/10	
Problem	5	6	7	8	**Total**
Points/Out of	/15	/15	/9	/15	

(1) Compute the integrals. Indicate which method of integration you are using.

$(a) \int x^2 e^x \, dx$

$(b) \int \cos^2(3x) \, dx$

(2) Compute the integrals. Indicate which method of integration you are using.

(a) $\displaystyle\int \frac{dx}{1-x^2}$

(b) $\displaystyle\int \frac{dx}{x\sqrt{1+x^2}}$.

(3) Determine if the **series** is convergent or divergent. Indicate which convergence test you are using.

(*a*) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n^2}{2n^2 + n}$

(*b*) $\displaystyle\sum_{n=1}^{\infty} \frac{n^n}{n! \cdot 4^n}$ Hint: You may need the limit $\displaystyle\lim_{n\to\infty} \left(\frac{n+1}{n}\right)^n = e \approx 2.71828...$

(4) Compute the improper integral or indicate why it diverges

$$\int_1^\infty (x^{-2} - e^{-x})dx$$

(5) Find the centroid of the region lying between the graphs of the functions $y = x^{1/3}$, and $y = x^3$ over the interval $[0, 1]$.
Hint: For finding y_{CM} you may use the formula $M_x = \frac{\rho}{2}\int_a^b (f_1(x)^2 - f_2(x)^2)dx$, where ρ is the density of the material.

(6) Find the radius of convergence and the interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{1}{n \cdot 2^n} \cdot (x-1)^n.$$

(7) Find the Maclaurin series of

$$f(x) = \frac{1}{1+x^2},$$

and the interval on which the expansion is valid.

(8) Find the area of the region bounded by the two circles

$$r = \sin\theta \quad \text{and} \quad r = 1/2.$$