NAME ______________________________ Rec. Instructor: __________________

Signature ______________________________ Rec. Time ____________

CALCULUS II - FINAL EXAM
May 11, 2016

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12	25	25	31	28	25	28	26	200

Show all work for full credit. No books, notes or calculators are permitted. The point value of each problem is given in the left-hand margin. You have 1 hour and 50 minutes.

(12) 1. Find the interval of convergence for the power series. Make clear the status of any endpoints.

$$\sum_{n=1}^{\infty} \frac{(x-2)^n}{n\, 3^n}.$$

Interval of convergence: ____________________.

2. Evaluate the following integrals.

(7) a) $\int x^2 \ln x \, dx$

(9) b) $\int \frac{dx}{(9+x^2)^{\frac{3}{2}}}$ (Hint: trig. substitution)

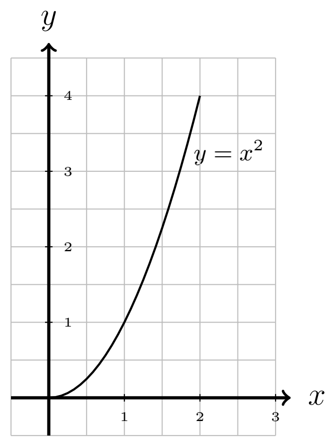
(9) c) $\int \frac{x+3}{x(x-3)} \, dx$ (Hint: partial fractions)

(8) d) $\int \sin^8 x \cos^3 x \; dx$

(8) e) $\int x^2 e^x \; dx$

(9) 3. Solve the initial value problem $\dfrac{dy}{dx} = 3x^2(1+y^2)$, $y(0) = 1$. Solve for y.

(18) 4. a) Set up, **but do not evaluate**, an integral representing the length of the curve $y = x^2$, $0 \le x \le 2$.



b) Set up **but do not evaluate** an integral representing the area of the surface obtained by revolving the curve in (a) about the x-axis.

c) Set up **but do not evaluate** an integral representing the volume of the solid obtained by revolving the region **under** the curve in (a) and above the x-axis about the y-axis.

5. Evaluate the following limits. Show all work.

(6) a) $\lim_{x \to 0} \dfrac{1 - \cos x}{x^2}$

(7) b) $\lim_{x \to \infty} (1 + 2x)^{1/x}$

6. Indicate whether the series converges or diverges (circle one). State which test you are using and implement the test as clearly as you can (5 of the 7 points are for the work!).

(7) a) $\displaystyle\sum_{n=2}^{\infty} \frac{5n^2}{\sqrt{n^7+5}}$ Converges Diverges. Test Used: ____________________

(7) b) $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{5^n}$ Converges Diverges. Test Used: ____________________

(7) c) $\displaystyle\sum_{n=3}^{\infty} \frac{1}{n\sqrt{\ln n}}$ Converges Diverges. Test Used: ____________________

(7) d) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{2n+1}{3n+1}\right)^n$ Converges Diverges. Test Used: ____________________

(7) 8. Find the equation of the tangent to the parametric curve

$$x = t - t^2, \quad y = 1 + t + t^2$$

at the point corresponding to $t = -1$.

(18) 9. a) Sketch the parametric curve (indicate the direction with an arrow)

$$x = 2\cos t, \quad y = \sin t, \qquad 0 \le t \le \frac{3\pi}{2}$$

b) Set up, **but do not evaluate,** an integral representing the length of the curve in (a).

c) Eliminate t to give an x, y equation for the curve in (a).

(10) 10. Find the first **four** non-zero terms of the Taylor series for $f(x) = \dfrac{1}{x^3}$ centered at $a = 1$.

(18) 11. Use known series to find the Maclaurin series through degree three:

a) $f(x) = \dfrac{1}{1+2x}$

b) $g(x) = e^x \cos x$

c) $h(x) = \sqrt{1+2x}$ (Hint: Binomial Theorem).

(6) 12. Determine whether the series $\sum_{n=0}^{\infty} \frac{2^{n+1}}{3^n}$ converges. If so find its sum.

(10) 13. a) Sketch the polar curve $r = 1 + \cos\theta$ for $0 \leq \theta \leq 2\pi$.

b) The point in (a) corresponding to $\theta = \pi/3$ has cartesian coordinates $(x, y) = \big(\_\_\_\_\_, \_\_\_\_\_\big)$.

(10) 14. a) Sketch the polar curve $r = 3\sin 4\theta$ for $0 \leq \theta \leq 2\pi$.

b) Set up, **but do not evaluate,** an integral representing the area of one petal of this curve.

$$\int \cos x \; dx = \sin x + C \qquad \int \sin x \; dx = -\cos x + C$$

$$\int \sec x \; dx = \ln|\sec x + \tan x| + C \qquad \int \tan x \; dx = -\ln|\cos x| + C$$

$$\int \sec x \tan x \; dx = \sec x + C \qquad \int \sec^2 x \; dx = \tan x + C$$

$$\int \csc x \; dx = \ln|\csc x - \cot x| + C \qquad \int \cot x \; dx = \ln|\sin x| + C$$

$$\int \csc x \cot x \; dx = -\csc x + C \qquad \int \csc^2 x \; dx = -\cot x + C$$

$$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\left(\frac{x}{a}\right) + C \qquad \int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\left(\frac{x}{a}\right) + C$$

$$\int \sqrt{a^2-u^2}\; du = \frac{1}{2}\left(u\sqrt{a^2-u^2} + a^2 \arcsin\left(\frac{u}{a}\right)\right) + C\,, \qquad \int \sinh x \; dx = \cosh x + C$$

$$\int \sqrt{u^2 \pm a^2}\; du = \frac{1}{2}\left(u\sqrt{u^2 \pm a^2} \pm a^2 \ln\left|u + \sqrt{u^2 \pm a^2}\right|\right) + C\,, \qquad \int \cosh x \; dx = \sinh x + C$$

$$\int \sin^n x \; dx = -\frac{\sin^{n-1} x \cos x}{n} + \frac{n-1}{n}\int \sin^{n-2} x \; dx, \qquad \int \operatorname{sech}^2 x \; dx = \tanh x + C$$

$$\int \cos^n x \; dx = \frac{\cos^{n-1} x \sin x}{n} + \frac{n-1}{n}\int \cos^{n-2} x \; dx, \qquad \int \tanh x \; dx = \ln\cosh x + C$$

$$\int \tan^n x \; dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x \; dx\,,\; n \neq 1\,, \qquad \int \operatorname{sech} x \; dx = \arctan|\sinh x| + C$$

$$\int \sec^n x \; dx = \frac{\sec^{n-2} x \tan x}{n-1} + \frac{n-2}{n-1}\int \sec^{n-2} x \; dx\,,\; n \neq 1\,,$$

$$\cos^2 x + \sin^2 x = 1, \qquad \sec^2 x = \tan^2 x + 1\,, \qquad \cosh^2 x - \sinh^2 x = 1$$

$$\sin^2 x = \tfrac{1}{2}(1 - \cos 2x), \qquad \cos^2 x = \tfrac{1}{2}(1 + \cos 2x), \qquad \sin 2x = 2\sin x \cos x$$

$$\sinh^2 x = \tfrac{1}{2}(\cosh 2x - 1), \qquad \cosh^2 x = \tfrac{1}{2}(\cosh 2x + 1), \qquad \sinh 2x = 2\sinh x \cosh x$$

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \qquad e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \qquad \ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^n}{n}$$

$$\arctan x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} \qquad \sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} \qquad \cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$$