Math 221 Calculus 2
Professor John Maginnis

Midterm Exam 1
September 19, 2017

Your name: Solutions

Rec. Instr.: ______________ Rec. Time: ______________

Show all your work in the space provided under each question. Please write neatly and present your answers in an organized way. You may use your one sheet of notes, but no books or calculators. This exam is worth 60 points. The chart below indicates how many points each problem is worth.

Problem	1	2	3	4
Points	/14	/9	/9	/10
Problem	5	6		Total
Points	/9	/9		/60

1. Evaluate the integrals.

(a)

$$\int x^2 \cos(x^3)\, dx$$

Substitution $u = x^3$

$du = 3x^2 dx$

$\frac{1}{3} du = x^2 dx$

$$\int \frac{1}{3}\cos(u)\, du = \frac{1}{3}\sin(u) + C$$

$$= \boxed{\frac{1}{3}\sin(x^3) + C}$$

(b)

$$\int x^2 \cos(x)\, dx$$

Integration by Parts (twice)

$u = x^2 \quad dv = \cos(x)\, dx$

$du = 2x\, dx \quad v = \sin(x)$

$$\int x^2 \cos(x)\, dx = x^2 \sin(x) - \int 2x \sin(x)\, dx$$

$u = -2x \quad dv = \sin(x)\, dx$

$du = -2\, dx \quad v = -\cos(x)$

$$\int x^2 \cos(x)\, dx = x^2 \sin(x) + 2x\cos(x) - \int 2\cos(x)\, dx$$

$$= \boxed{x^2 \sin(x) + 2x\cos(x) - 2\sin(x) + C}$$

2. Use a substitution to evaluate the integral.

$$\int \frac{\sin^3(x)}{\cos^4(x)}\,dx$$

$u = \cos(x)$

$du = -\sin(x)\,dx$

Also need Pythagorean's Theorem

$\sin^2(x) = 1 - \cos^2(x) = 1 - u^2$

$$\int \frac{\sin^3(x)}{\cos^4(x)}\,dx = \int \frac{(1-\cos^2(x))\sin(x)}{\cos^4(x)}\,dx$$

$$= \int \frac{(1-u^2)(-du)}{u^4} = \int \frac{-1+u^2}{u^4}\,du$$

$$= \int -u^{-4} + u^{-2}\,du = \frac{-u^{-3}}{-3} + \frac{u^{-1}}{-1} + C$$

$$= \boxed{\frac{1}{3\cos^3(x)} - \frac{1}{\cos(x)} + C}$$

3. Use partial fractions to evaluate the integral.

$$\int \frac{x+12}{x^3+3x^2}\,dx \qquad x^3+3x^2 = x^2(x+3)$$

$$\frac{x+12}{x^3+3x^2} = \frac{Ax+B}{x^2} + \frac{C}{x+3} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+3}$$

$$x+12 = Ax(x+3) + B(x+3) + Cx^2$$

If $x=0$ then $12 = 3B$, so $B=4$.

If $x=-3$ then $9 = 9C$, so $C=1$.

Coefficients of x^2 yields $0 = A+C$, $A=-C=-1$.

$$\int \frac{x+12}{x^3+3x^2}\,dx = \int\left(\frac{-1}{x} + \frac{4}{x^2} + \frac{1}{x+3}\right)dx$$

$$= \boxed{-\ln|x| - \frac{4}{x} + \ln|x+3| + C}$$

$$= \ln\left|\frac{x+3}{x}\right| - \frac{4}{x} + C.$$

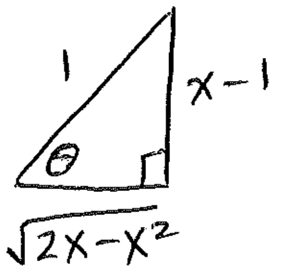
4. Use a trigonometric substitution to evaluate the integral.

Hint: $2x - x^2 = 1 - (x-1)^2$.

$$\int \sqrt{2x - x^2}\, dx$$

Either substitute $u = x - 1$ first, or

just use



so that $x - 1 = \sin(\theta)$

$$dx = \cos(\theta)\, d\theta$$

and $\sqrt{2X - X^2} = \cos(\theta)$

$$\int \sqrt{2X - X^2}\, dx = \int \cos\theta \cdot \cos\theta \, d\theta = \int \cos^2(\theta)\, d\theta$$

$$= \frac{\theta}{2} + \frac{\sin(2\theta)}{4} + C = \frac{\theta}{2} + \frac{\sin(\theta)\cos(\theta)}{2} + C$$

$$= \boxed{\frac{1}{2}\sin^{-1}(x-1) + \frac{1}{2}(x-1)\cdot\sqrt{2X - X^2} + C}$$

5. Write the improper integral as a limit, and evaluate.

$$\int_0^1 \frac{x^4}{\sqrt[3]{1-x^5}}\,dx$$

Substitution $u = 1 - x^5$

$du = -5x^4\,dx$

$-\frac{1}{5}\,du = x^4\,dx$

$$\lim_{R \to 1} \left(\int_0^R \frac{x^4}{\sqrt[3]{1-x^5}}\,dx \right)$$

$$\int \frac{x^4}{\sqrt[3]{1-x^5}}\,dx = \frac{-1}{5} \int \frac{du}{\sqrt[3]{u}}$$

$$= \frac{-1}{5}\left(\frac{u^{2/3}}{2/3}\right) + C$$

$$= \frac{-3}{10}(1-x^5)^{2/3} + C$$

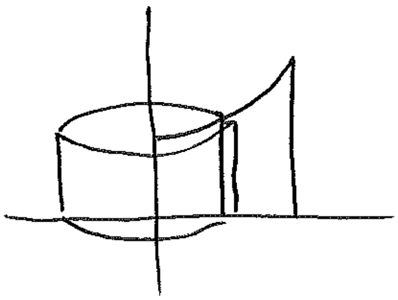
$$\lim_{R \to 1} \left[\frac{-3}{10}(1-x^5)^{2/3} \right]_0^R$$

$$= \lim_{R \to 1} \left[\frac{-3}{10}(1-R^5)^{2/3} - \left(\frac{-3}{10}\right) \right]$$

$$= 0 + \frac{3}{10} = \boxed{\frac{3}{10}}$$

6. Find the volume of revolution formed by revolving around the y-axis the region under the curve.

$y = \sqrt{x^2+1}$ for $0 \le x \le 1$.



Cylindrical Shells $dV = 2\pi R H \, dx$

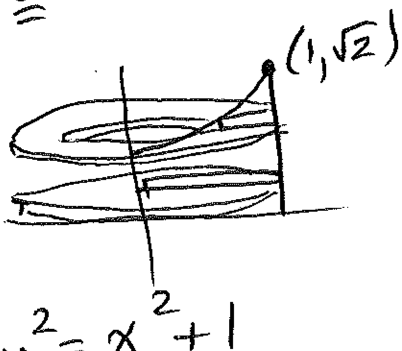
$R = x, \quad H = y = \sqrt{x^2+1}$

$$\int_0^1 2\pi x \sqrt{x^2+1} \, dx = \pi \int_1^2 \sqrt{u} \, du = \pi \left[\frac{2}{3} u^{3/2} \right]_1^2$$

Substitution $u = x^2+1$
$du = 2x \, dx$

$$= \boxed{\frac{2\pi}{3}\left(2\sqrt{2} - 1\right)}$$

or could use disks/washers $dV = \pi(R^2 - r^2)\,dy$



$$\int_0^1 \pi \, dy + \int_1^{\sqrt{2}} \pi(1 - x^2)\,dy$$

$y^2 = x^2 + 1$
$1 - x^2 = 2 - y^2$

$$[\pi y]_0^1 + \int_1^{\sqrt{2}} \pi(2 - y^2)\,dy$$

$$\pi + \pi\left[2y - \frac{y^3}{3}\right]_1^{\sqrt{2}} = \pi + \pi\left(2\sqrt{2} - \frac{2\sqrt{2}}{3} - \left(2 - \frac{1}{3}\right)\right)$$

$$= \boxed{\frac{4\sqrt{2}}{3}\pi - \frac{2}{3}\pi}$$