Math 221 Calculus 2
Professor John Maginnis

Midterm Exam 1
January 31, 2017

Your name: Solutions

Rec. Instr.: ____________ Rec. Time: ____________

Show all your work in the space provided under each question. Please write neatly and present your answers in an organized way. You may use your one sheet of notes, but no books or calculators. This exam is worth 60 points. The chart below indicates how many points each problem is worth.

Problem	1	2	3	4
Points	/15	/7	/8	/7
Problem	5	6	7	Total
Points	/8	/7	/8	/60

1. (a) Use the substitution $u = x^2 - 1$ to evaluate the integral.

$$\int \frac{x}{\sqrt{x^2-1}}\,dx = \int \frac{\frac{1}{2}du}{\sqrt{u}} = \frac{1}{2}\int u^{-1/2}\,du$$

Substitution $u = x^2 - 1$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$$= \frac{1}{2}\cdot\frac{u^{1/2}}{1/2} + C$$

$$= \sqrt{u} + C$$

$$= \boxed{\sqrt{x^2-1} + C}$$

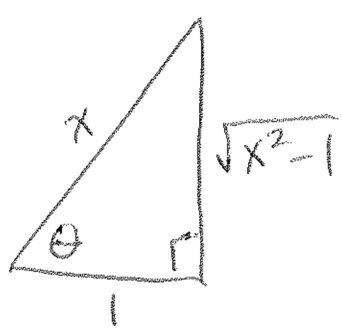
(b) Use the trigonometric substitution $x = \sec(\theta)$ to evaluate the integral.

$$\int \frac{x}{\sqrt{x^2-1}}\,dx = \int \frac{\sec(\theta)}{\tan(\theta)}\cdot\sec(\theta)\tan(\theta)\,d\theta$$

$$= \int \sec^2(\theta)\,d\theta$$

$$= \tan(\theta) + C$$

$$= \boxed{\sqrt{x^2-1} + C}$$



$x = \sec(\theta)$

$dx = \sec(\theta)\tan(\theta)\,d\theta$

$\sqrt{x^2-1} = \tan(\theta)$

2. Evaluate the integral.

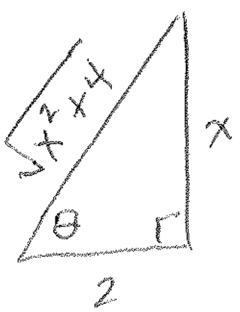
$$\int \tan^2(5x)\,dx = \int \left(\sec^2(5x) - 1\right) dx$$

$$= \boxed{\frac{1}{5}\tan(5x) - x + C}$$

First use the Pythagorean Theorem,
then $u = 5x$, $du = 5\,dx$, $\frac{1}{5}du = dx$.

3. Use a trigonometric substitution to evaluate the integral.

$$\int \frac{dx}{(x^2+4)^2} = \int \frac{2\sec^2(\theta)\,d\theta}{(2\sec(\theta))^4} = \frac{1}{8}\int \frac{d\theta}{\sec^2\theta}$$



$$= \frac{1}{8}\int \cos^2(\theta)\,d\theta$$

$$= \frac{1}{8}\left(\frac{\theta}{2} + \frac{\sin(\theta)\cos(\theta)}{2}\right) + C$$

$x = 2\tan(\theta)$

$dx = 2\sec^2(\theta)\,d\theta$

$\sqrt{x^2+4} = 2\sec(\theta)$

$$= \frac{1}{16}\tan^{-1}\left(\frac{x}{2}\right) + \frac{1}{16}\cdot\frac{x}{\sqrt{x^2+4}}\cdot\frac{2}{\sqrt{x^2+4}} + C$$

$$= \boxed{\frac{1}{16}\tan^{-1}\left(\frac{x}{2}\right) + \frac{x}{8(x^2+4)} + C}$$

Note $\cos^2(\theta) = \dfrac{1+\cos(2\theta)}{2}$, and $\dfrac{\theta}{2} + \dfrac{\sin(2\theta)}{4} = \dfrac{\theta}{2} + \dfrac{2\sin\theta\cos\theta}{4}$.

4. Use integration by parts twice to evaluate the integral.

$$\int x^2 \sin(3x)\, dx = \frac{-x^2}{3}\cos(3x) - \int \frac{-2x}{3}\cos(3x)\, dx$$

$u = x^2$, $dv = \sin(3x)\,dx$
$du = 2x\,dx$, $v = -\frac{1}{3}\cos(3x)$

$u = \frac{2x}{3}$, $dv = \cos(3x)\,dx$
$du = \frac{2}{3}\,dx$, $v = \frac{1}{3}\sin(3x)$

$$= \frac{-x^2}{3}\cos(3x) + \frac{2x}{9}\sin(3x) - \int \frac{2}{9}\sin(3x)\,dx$$

$$= \boxed{\frac{-x^2}{3}\cos(3x) + \frac{2x}{9}\sin(3x) + \frac{2}{27}\cos(3x) + C}$$

5. Evaluate the integral. Use substitution $u = \sin(x)$.

$$\int \sin^5(x)\cos^3(x)\, dx = \int \sin^5 x \cos^2 x \cos x\, dx$$

$u = \sin x$
$du = \cos x\, dx$
$\cos^2 x = 1 - \sin^2 x = 1 - u^2$

$$= \int u^5(1-u^2)\,du = \int u^5 - u^7\,du$$

$$= \frac{1}{6}u^6 - \frac{1}{8}u^8 + C$$

$$= \boxed{\frac{1}{6}\sin^6(x) - \frac{1}{8}\sin^8(x) + C}$$

Note $u = \cos x$
leads to $\int -(1-u^2)^2 u^3\,du = -\frac{1}{4}\cos^4(x) + \frac{1}{3}\cos^6(x) - \frac{1}{8}\cos^8(x) + C.$

6. Evaluate the definite integral.

$$\int_1^e \frac{(\ln(x))^2}{x}\,dx = \int_0^1 u^2\,du = \left[\frac{1}{3}u^3\right]_0^1$$

$$= \frac{1}{3} - 0 = \boxed{\frac{1}{3}}$$

Substitution

$u = \ln(x)$

$du = \frac{1}{x}\,dx$

Note $\left[\frac{1}{3}(\ln x)^3\right]_1^e = \frac{1}{3}$

7. Evaluate the integral.

$$\int \frac{\ln(x)}{x^2}\,dx = \frac{-\ln(x)}{x} - \int \frac{-1}{x^2}\,dx$$

Integration by Parts

$u = \ln(x)$ $\quad dv = \frac{1}{x^2}\,dx$

$du = \frac{1}{x}\,dx$ $\quad v = \frac{-1}{x}$

$$\boxed{\frac{-\ln(x)}{x} - \frac{1}{x} + C}$$

Note a substitution $u = \ln(x)$ leads to $\int \frac{u}{e^u}\,du$, which by integration by parts is $\frac{-u}{e^u} - \frac{1}{e^u} + C = \frac{-\ln x}{x} - \frac{1}{x} + C$.