

Your name:\_\_\_\_\_

Rec. Instructor:\_\_\_\_\_

Rec. Time:\_\_\_\_\_

Show all your work in the space provided under each question. Please write neatly and present your answers in an organized way. You may use one sheet of notes, but no books, calculators, cell phones, or other electronic devices. This exam is worth 120 points. The chart below indicates how many points each problem is worth.

Problem	1	2	3	4
Points	/12	/12	/12	/12
Problem	5	6	7	8
Points	/14	/8	/10	/12
Problem	9	10		Total
Points	/16	/12		/120

1. Evaluate the definite integrals.

(a)

$$\int_1^e x \ln x \, dx$$

(b) Write the improper integral as a limit, and evaluate.

$$\int_0^1 \frac{e^{\sqrt{x}}}{\sqrt{x}} \, dx$$

2. Evaluate the integrals.

(a)

$$\int \cos(\sqrt{x}) \, dx$$

Hint: Use substitution  $u = \sqrt{x}$ .

(b)

$$\int \frac{2}{x(x^2 + 1)} \, dx$$

3. Determine whether the given positive series converges or diverges. List each test of convergence used.

(a)

$$\sum_{n=0}^{\infty} \frac{n^3}{n^3 + 1}$$

(b)

$$\sum_{n=1}^{\infty} \frac{1}{n} - \frac{1}{n+1}$$

4. Determine whether the series converges or diverges. List each test of convergence used.

(a)

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{(n+2)^{\frac{1}{3}}}$$

(b)

$$\sum_{n=2}^{\infty} a_n = -\frac{1}{2(2^2)} - \frac{1}{3(2^3)} + \frac{1}{4(2^4)} + \frac{1}{5(2^5)} - \frac{1}{6(2^6)} - \frac{1}{7(2^7)} + \frac{1}{8(2^8)} + \frac{1}{9(2^9)} \dots$$

5. Find the fourth Taylor polynomial  $p_4(x)$  centered at  $c = -1$  for the function

$$f(x) = \ln(x + 2).$$

6. Determine whether the sequence  $\{a_n\}_{n=1}^{\infty}$  converges. If the sequence converges, calculate its limit.

$$a_n = n \sin\left(\frac{1}{n}\right)$$

7. A circle is given in Cartesian coordinates by

$$(x - 2)^2 + y^2 = 4.$$

Find a polar equation for the circle. Write your solution in the form  $r = f(\theta)$ .

8. Find the arc length of the parametric curve

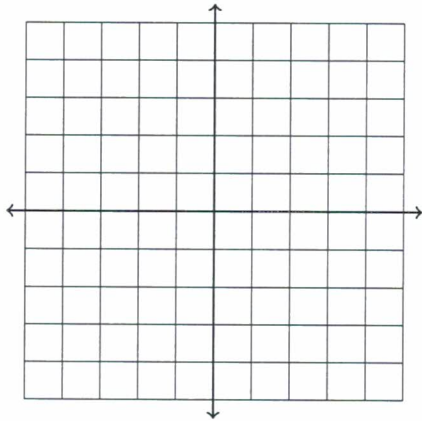
$$x(t) = e^t \cos t, \quad y(t) = e^t \sin t, \quad t \in [0, 1].$$



9. Consider the polar curve

$$r = 4 \cos(2\theta).$$

(a) Graph the curve, plotting the points where  $\theta = 0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3},$  and  $\frac{\pi}{2}$ .



(b) Find the area of the region enclosed by this polar curve.

10. An ellipse is given by the equation

$$\frac{x^2}{4} + \frac{y^2}{9} = 1.$$

(a) Find parametric equations for the ellipse of the form  $x = x(t)$ ,  $y = y(t)$ ,  $t \in [0, 2\pi]$ .

(b) Graph the parametric equations found in part (a) from  $t = 0$  to  $t = \frac{\pi}{2}$ .

