Name ______________________________ Rec. Instr. ______________________
Signature ____________________________ Rec. Time ______________________

Math 221 – Final Exam – May 9, 2018

No books, calculators, or notes are allowed. Please make sure that your cell phone is turned off. You will have 110 minutes to complete the exam.

SHOW YOUR WORK!

Problem	Points	Points Possible	Problem	Points	Points Possible
1		25	7		12
2		5	8		5
3		6	9		5
4		6	10		4
5		15	11		6
6		5	12		6

Total Score

$\sin(ax)\sin(bx) = \frac{1}{2}\cos((a-b)x) - \frac{1}{2}\cos((a+b)x)$

$\cos(ax)\cos(bx) = \frac{1}{2}\cos((a-b)x) + \frac{1}{2}\cos((a+b)x)$

$\sin(ax)\cos(bx) = \frac{1}{2}\sin((a-b)x) + \frac{1}{2}\sin((a+b)x)$

$$\sin^2(x) = \frac{1-\cos(2x)}{2} \qquad \cos^2(x) = \frac{1+\cos(2x)}{2}$$

$$\int \tan(x)\,dx = \ln|\sec(x)| + C \qquad \int \sec(x)\,dx = \ln|\sec(x)+\tan(x)| + C$$

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\left(\frac{x}{a}\right) + C \qquad \int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\left(\frac{x}{a}\right) + C$$

$$\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\text{arcsec}\left(\frac{x}{a}\right) + C$$

$$\int \sin^n(x)\,dx = -\frac{\sin^{n-1}(x)\cos(x)}{n} + \frac{n-1}{n}\int \sin^{n-2}(x)\,dx$$

$$\int \cos^n(x)\,dx = \frac{\cos^{n-1}(x)\sin(x)}{n} + \frac{n-1}{n}\int \cos^{n-2}(x)\,dx$$

$$\int \tan^n(x)\,dx = \frac{\tan^{n-1}(x)}{n-1} - \int \tan^{n-2}(x)\,dx$$

$$\int \sec^n(x)\,dx = \frac{\sec^{n-2}(x)\tan(x)}{n-1} + \frac{n-2}{n-1}\int \sec^{n-2}(x)\,dx$$

$$\text{Arc Length} = \int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx \qquad \text{Surface Area} = \int_a^b 2\pi y\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

$$M_x = \frac{\rho}{2}\int_a^b \left(f(x)^2 - g(x)^2\right)\,dx \qquad M_y = \rho\int_a^b x\left(f(x)-g(x)\right)\,dx$$

$$T_N = \frac{\Delta x}{2}\left(f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{N-1}) + f(x_N)\right)$$

$$S_N = \frac{\Delta x}{3}\left(f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + 2f(x_4) + \cdots + 2f(x_{N-2}) + 4f(x_{N-1}) + f(x_N)\right)$$

$$\text{Work} = \int_a^b F(x)\,dx \qquad \cosh^2(x) - \sinh^2(x) = 1 \qquad \frac{d}{dx}\sinh(x) = \cosh(x)$$

$$\frac{d}{dx}\cosh(x) = \sinh(x) \qquad \frac{d}{dx}\text{csch}(x) = -\text{csch}(x)\text{coth}(x) \qquad \frac{d}{dx}\text{sech}(x) = -\text{sech}(x)\text{tanh}(x)$$

$$\frac{d}{dx}\text{tanh}(x) = \text{sech}^2(x) \qquad \frac{d}{dx}\text{coth}(x) = -\text{csch}^2(x) \qquad \frac{d}{dx}\sinh^{-1}(x) = \frac{1}{\sqrt{1+x^2}}$$

$$\frac{d}{dx}\cosh^{-1}(x) = \frac{1}{\sqrt{x^2-1}} \qquad \frac{d}{dx}\text{csch}^{-1}(x) = \frac{-1}{|x|\sqrt{1+x^2}} \qquad \frac{d}{dx}\text{sech}^{-1}(x) = \frac{-1}{x\sqrt{1-x^2}}$$

$$\frac{d}{dx}\text{tanh}^{-1}(x) = \frac{1}{1-x^2} \qquad \frac{d}{dx}\text{coth}^{-1}(x) = \frac{1}{1-x^2} \qquad \frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$$

$$e^x = \sum_{n=0}^{\infty}\frac{x^n}{n!} \qquad \sin(x) = \sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+1}}{(2n+1)!} \qquad \cos(x) = \sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}}{(2n)!}$$

$$\ln(1+x) = \sum_{n=1}^{\infty}\frac{(-1)^{n+1}x^n}{n} \qquad \arctan(x) = \sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+1}}{2n+1} \qquad (1+x)^r = \sum_{n=0}^{\infty}\binom{r}{n}x^n$$

$$\text{Arc Length} = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt \qquad \text{Area} = \pm\int_a^b y(t)x'(t)\,dt$$

$$\text{Arc Length} = \int_\alpha^\beta \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\,d\theta \qquad \text{Area} = \frac{1}{2}\int_\alpha^\beta r^2\,d\theta$$

1. (5 points each) Evaluate the following:

A. $\dfrac{d}{dx}\left(\tanh^{-1}(x^2)\cdot\sinh(x^5)\right)$

B. $\displaystyle\int x^2\ln(x)\,dx$

C. $\displaystyle\int \frac{1}{x^2+x}dx$

D. $\displaystyle\sum_{n=0}^{\infty} \frac{2^n + 4}{3^n}$

E. $\displaystyle\int \sin^4(\theta)\cos^3(\theta)\, d\theta$

2. (5 points) Find the degree-2 Maclaurin polynomial of $f(x) = \ln(1 + 2x)$.

3. (6 points) Find the general solution to $\dfrac{dy}{dx} = (2x+1)y$.

4. (6 points) Use the integral test to determine if $\displaystyle\sum_{n=1}^{\infty} 2ne^{-n^2}$ converges or diverges. (Use proper limit notation.)

5. (5 points each) Determine if each of the series below converge or diverge. Make sure to write the names of any convergence tests you use and justify why the convergence tests apply.

A. $\displaystyle\sum_{n=1}^{\infty} \frac{3n^5 + n}{n^5 + 7}$

B. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$

C. $\displaystyle\sum_{n=1}^{\infty} \frac{2n + 8}{5n^3 + 7}$

6. (5 points) Determine the interval of convergence for the power series $\displaystyle\sum_{n=0}^{\infty} \frac{5^n(x+3)^n}{n!}$.

7. (6 points each) Find the Maclaurin series of:

A. $\dfrac{x}{1-x^2}$

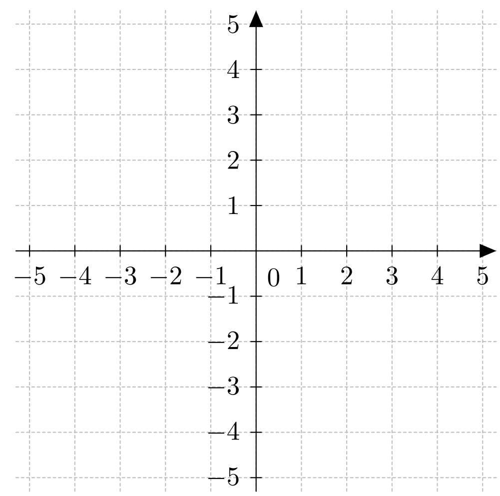
B. $\displaystyle\int \frac{\sin(x)}{x}\,dx$

8. (5 points) Find the arc length of $x(t) = e^t + 3$, $y(t) = 2e^t + 5$ for $0 \le t \le 1$.

9. (5 points) Find the area bounded by the x-axis and the parametric curve $x(t) = \cos(t)$, $y(t) = 2\sin(t)$ for $0 \le t \le \pi$.

10. (4 points) **Write** an integral that calculates the arc length of $r = \theta^2$ from $\theta = 0$ to $\theta = 5$. **You do not need to evaluate the integral!**

11. (6 points) Convert $r = 4\sin(\theta)$ to an equation in Cartesian (rectangular) coordinates. State what geometric shape it is, and graph the equation.



12. (6 points) Below, the polar equation $r = 4\sin(2\theta)$ is graphed. Find the area enclosed by one petal.