

# Math 221 FINAL EXAM

July 28, 2017

Student's Name:

Instructor's Name:

Q.N.	1	2	3	4	5	Total
Points	/25	/25	/20	/25	/25	/120

Show all work in detail for full credit. No books and calculators are permitted. Use the back page as a sketch paper.

1.

(a) (10 points) Evaluate  $\int \cos^3 x \sin^2 x \, dx$

(b) (15 points) Evaluate  $\int_0^3 \frac{1}{\sqrt{3-x}} \, dx$ , if possible. State whether the integral converges or diverges.

2.

- (a) (10 points). Define  $R$  as the region bounded above by the graph of  $f(x) = 3x - x^2$  and below by the  $x$ -axis over the interval  $[0, 2]$ . Find the volume of the solid of revolution formed by revolving  $R$  around the  $y$ -axis.

- (b) (15 points) Let  $f(x) = \sqrt{1-x}$  over the interval  $[0, 1/2]$ . Find the surface area of the surface generated by revolving the graph of  $f(x)$  around the  $x$ -axis.

3.

- (a) (10 points). Suppose it takes a force of 8 N to stretch a spring 6 inch from the equilibrium position. How much work is done to stretch the spring 1 ft from the equilibrium position?

- (b) (15 points). Test whether the following two series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{(2n)!}{n^{2n}}$$

4 (20 points). Use the series for  $f(x) = \frac{1}{1-x}$  on  $|x| < 1$  to construct a series for

$$\frac{1}{(1-x)(3+x)}.$$

Determine the interval of convergence.

5.

- (a) (15 points). Find the fourth degree Taylor polynomials for  $f(x) = \sqrt{x}$  at  $x = 1$ . Use this polynomial to estimate  $\sqrt{6}$  and bound the error.

- (b) (10 points). Rewrite the following polar equation into rectangular coordinates and sketch the graph. Also identity the symmetry of this graph.

$$r = 4\sin\theta$$