Name: _____

Recitation time: _____   Rec. instructor: _____

MATH 221 - Midterm 2
October 22, 2019

- This exam contains 8 pages (including this cover page) and 7 questions.
- No books, calculators, or notes are allowed. You must show all your work to get credit for your answers.
- You have 1 hour and 15 minutes to complete the exam.

Question	Points	Score
1	14	
2	14	
3	15	
4	14	
5	14	
6	15	
7	14	
Total:	100	

$$\cosh^2(x) - \sinh^2(x) = 1, \quad \cosh^2(x) = \frac{1 + \cosh(2x)}{2}, \quad \sinh(2x) = 2\sinh(x)\cosh(x)$$

$$\frac{d}{dx}(\sinh x) = \cosh x, \quad \frac{d}{dx}(\cosh x) = \sinh x, \quad \frac{d}{dx}(\tanh x) = \text{sech}^2 x$$

$$\frac{d}{dx}(\coth x) = -\text{csch}^2 x, \quad \frac{d}{dx}(\text{sech} x) = -\text{sech} x \tanh x, \quad \frac{d}{dx}(\text{csch} x) = -\text{csch} x \coth x$$

$$\frac{d}{dx}(\sinh^{-1} x) = \frac{1}{\sqrt{1 + x^2}}, \quad \frac{d}{dx}(\cosh^{-1} x) = \frac{1}{\sqrt{x^2 - 1}}, \quad \frac{d}{dx}(\tanh^{-1} x) = \frac{1}{1 - x^2}$$

$$\frac{d}{dx}(\text{sech}^{-1} x) = \frac{-1}{x\sqrt{1 - x^2}}, \quad \frac{d}{dx}(\coth^{-1} x) = \frac{1}{1 - x^2}, \quad \frac{d}{dx}(\text{csch}^{-1} x) = \frac{-1}{|x|\sqrt{1 + x^2}}$$

$$M_x = \frac{\rho}{2} \int_a^b f(x)^2 - g(x)^2 \, dx \qquad M_y = \rho \int_a^b x(f(x) - g(x)) \, dx$$

1. (a) (6 points) Write an integral that calculates the length of the curve $y = x + \sin x$, $0 \leq x \leq \pi$. **Do not evaluate the integral.**

(b) (8 points) Find the surface area of the surface obtained by rotating the curve $y = \sqrt{x}$, $1 \leq x \leq 4$ around the x-axis. **Evaluate the integral.**

2. (14 points) Find the center of mass (centroid) $(\bar{x}, \bar{y})$ of the region bounded by $y = e^{-x}$, $x = 0$, $x = 1$ and the x-axis.

3. (a) (7 points) A spring requires $10\,J$ to stretch it $2\,m$ from its rest length. How much work is required to stretch the spring from $2\,m$ to $4\,m$ from its rest length?

    (b) (8 points) Find the work required to pump all the liquid out of a cylindrical tank that has a base of radius $10\,ft$ and height $50\,ft$. Use the fact that the density of the liquid is $\rho\,lb/ft^3$.

4. Evaluate the following

(a) (7 points) $\dfrac{d}{dx}\tanh^{-1}(\sin(x^2))$, where $\tanh^{-1}$ is the inverse function of $\tanh$.

(b) (7 points) $\displaystyle\int \dfrac{1}{\sqrt{9+x^2}}\,dx$, using the substitution $x = 3\sinh\theta$.

5. Consider the differential equation

$$\frac{dy}{dx} = x^3 y^2.$$

(a) (3 points) Find the constant solutions.

(b) (8 points) Find the general solution to the differential equation

(c) (3 points) Find the particular solution satisfying $y(0) = 2$.

6. (a) (7 points) Evaluate the limit of the sequence $\lim_{n} \dfrac{\ln(n)}{n^2}$.

(b) (8 points) Use the squeeze theorem to calculate $\lim_{n} \dfrac{2n - \cos(n)}{n}$.

7. Evaluate the series:

(a) (7 points) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n 5 + 2^n}{3^n}$

(b) (7 points) $\displaystyle\sum_{n=3}^{\infty} \frac{1}{n(n-1)}$