Name: _____

Recitation time: _____     Rec. instructor: _____

## MATH 221 - Midterm 3
## November 19, 2019

- This exam contains 8 pages (including this cover page) and 7 questions.
- No books, calculators, or notes are allowed. You must show all your work to get credit for your answers.
- You have 1 hour and 15 minutes to complete the exam.

| Question | Points | Score |
|:---:|:---:|:---:|
| 1 | 16 | |
| 2 | 16 | |
| 3 | 11 | |
| 4 | 16 | |
| 5 | 16 | |
| 6 | 16 | |
| 7 | 9 | |
| Total: | 100 | |

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$$

1. Determine whether the following series converge or diverge. Show all work to justify your answers.

   (a) (8 points) $\displaystyle\sum_{n=1}^{\infty} \frac{3n^2 - 5}{n^4 - n^2 + 1}$

   (b) (8 points) $\displaystyle\sum_{n=1}^{\infty} \sqrt[n]{n}$

2. Determine whether the following series converge or diverge. Show all work to justify your answers.

(a) (8 points) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\ln(n)}$

(b) (8 points) $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{(n+1)^n}$

3. (a) (5 points) Show the following series converges

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{3n+5}$$

(b) (6 points) Find the minimum $M$ that guarantees

$$\left| \sum_{n=1}^{\infty} \frac{(-1)^n}{3n+5} - \sum_{n=1}^{M} \frac{(-1)^n}{3n+5} \right| < 0.01$$

4. Determine whether the following series converge conditionally, converge absolutely, or diverge. Justify your answer.

(a) (8 points) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \cos(n)}{2^n}$

(b) (8 points) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n}{n+1}$

5. Determine for which values of $x$ the following power series converge.

(a) (8 points) $\displaystyle\sum_{n=1}^{\infty} \frac{5x^n}{(n+5)n!}$

(b) (8 points) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2(x-1)^n}{3^n}$

6.  (a) (8 points) Find the power series for the function $f(x) = \dfrac{2}{1+x^3}$. Determine the interval of convergence of the series.

   (b) (8 points) Consider the power series $g(x) = \displaystyle\sum_{n=1}^{\infty} \dfrac{x^n}{n}$. Find the power series for $\displaystyle\int \dfrac{g(x)}{x}\,dx$.

7. (9 points) Find the degree three Taylor polynomial of

$$f(x) = \ln(1 - x)$$

centered at $x = 0$.