NAME ______________________ Rec. Instructor: ______________

Signature ______________________ Rec. Time __________

CALCULUS II - EXAM 3

April 9, 2019

Show all work for full credit. No books, notes or calculators are permitted. The point value of each problem is given in the left-hand margin. You have 75 minutes.

Problem	Points	Possible	Problem	Points	Possible
1a		8	5		7
1b		8	6a		7
2		10	6b		7
3a		6	6c		7
3b		6	7		6
4a		6	8a		7
4b		8	8b		7
			Total Score		100

You are free to use the following formulas on any of the problems.

$\cosh(x) = \frac{1}{2}(e^x + e^{-x})$, $\sinh(x) = \frac{1}{2}(e^x - e^{-x})$,

$\cosh^2 x - \sinh^2 x = 1$, $\cosh^2 x = \frac{1}{2}(1 + \cosh(2x))$, $\sinh(2x) = 2\sinh(x)\cosh(x)$

$\frac{d}{dx}\sinh(x) = \cosh(x)$, $\frac{d}{dx}\cosh(x) = \sinh(x)$

$\frac{d}{dx}\tanh(x) = \text{sech}^2 x$, $\frac{d}{dx}\text{sech}(x) = -\text{sech}(x)\tanh(x)$.

$\frac{d}{dx}\sinh^{-1}(x) = \frac{1}{\sqrt{1+x^2}}$, $\frac{d}{dx}\cosh^{-1}(x) = \frac{1}{\sqrt{x^2-1}}$,

$\frac{d}{dx}\tanh^{-1}(x) = \frac{1}{1-x^2}$, $\frac{d}{dx}\text{sech}^{-1}(x) = \frac{-1}{x\sqrt{1-x^2}}$

1. Evaluate the following.

(8) a) $\dfrac{d}{dx}\, e^{2x} \sinh^{-1}(\sqrt{x})$, where $\sinh^{-1}$ is the inverse sinh function.

(8) b) $\displaystyle\int \sqrt{1+x^2}\, dx$, using the substitution $x = \sinh(\theta)$.

2. Consider the differential equation

$$\frac{dy}{dx} = \frac{\ln(x)}{xy^2}, \quad (x > 0).$$

(8) a) Find the general solution.

(2) b) Find the solution satisfying the initial condition $y(1) = 4$.

3. Find the limit of the sequence or state that it diverges.

(6) a) $\displaystyle\lim_{n\to\infty} \frac{(\ln(n))^2}{n}$

(6) b) $\displaystyle\lim_{n\to\infty} n\sin(2/n)$

4. Evaluate the series.

(6) a. $\displaystyle\sum_{n=2}^{\infty}(-1)^n\left(\frac{2}{3}\right)^n$

(8) b. $\displaystyle\sum_{n=3}^{\infty}\frac{2}{n^2-1}$

(7) **5.** Use the limit comparison test to determine whether the following series converges or diverges. Show all work to justify your answer.

$$\sum_{n=1}^{\infty}\frac{n^2+8}{4n^4-n^2}$$

6. Determine whether the following series converge or diverge. Show all work to justify your answers.

(7) a. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$.

(7) b. $\displaystyle\sum_{n=1}^{\infty} \cos(1/n)$.

(7) c. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n \ln(n)}$.

(6) **7.** Find the minimum M that guarantees that

$$\left|\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n-1} - \sum_{n=1}^{M} \frac{(-1)^{n+1}}{2n-1}\right| < .01.$$

8. Determine whether the following series converge conditionally, converge absolutely, or diverge. Justify your answer.

(7) a. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$.

(7) b. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}\sin(n)}{5e^n}$.