NAME ____________________ Rec. Instructor: ____________

Signature ____________________ Rec. Time ________

CALCULUS II - FINAL EXAM
May 15, 2019, 6:20 to 8:10 p.m.

Show all work for full credit. No books, notes or calculators are permitted. The point value of each problem is given in the left-hand margin. You have 1 hour and 50 minutes.

Problem	Points	Points Possible	Problem	Points	Points Possible
1a		10	8		12
1b		12	9		18
2a		10	10		12
2b		12	11		10
3		10	12		8
4		16	13		12
5		8	14		18
6		10	15		10
7		12			
			Total Score		200

$\frac{d}{dx}\tan x = \sec^2 x$ $\frac{d}{dx}\sec x = \sec x \tan x$ $\frac{d}{dx}b^x = b^x \ln(b)$

$\frac{d}{dx}\sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}}$ $\frac{d}{dx}\tan^{-1}(x) = \frac{1}{1+x^2}$ $\frac{d}{dx}\sec^{-1}(x) = \frac{1}{|x|\sqrt{x^2-1}}$

$$\int \tan x \; dx = -\ln|\cos x| + C \qquad \int \sec x \; dx = \ln|\sec x + \tan x| + C$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}(\frac{x}{a}) + C \qquad \int \frac{dx}{a^2 + x^2} = \frac{1}{a}\tan^{-1}(\frac{x}{a}) + C$$

$$\int \sin^n x \; dx = -\frac{\sin^{n-1} x \cos x}{n} + \frac{n-1}{n}\int \sin^{n-2} x \; dx,$$

$$\int \cos^n x \; dx = \frac{\cos^{n-1} x \sin x}{n} + \frac{n-1}{n}\int \cos^{n-2} x \; dx,$$

$$\int \tan^n x \; dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x \; dx \, ,$$

$$\int \sec^n x \; dx = \frac{\sec^{n-2} x \tan x}{n-1} + \frac{n-2}{n-1}\int \sec^{n-2} x \; dx$$

$M_x = \frac{1}{2}\int_a^b f(x)^2 - g(x)^2 \; dx \, ,\qquad M_y = \int_a^b x(f(x) - g(x)) \; dx \, .$

$|R_n(x)| \leq \frac{K}{(n+1)!}|x-a|^{n+1}$, with $K = \max_{a \leq c \leq x} |f^{(n+1)}(c)| \, .$

$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \, ,\qquad e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!},\qquad \ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}x^n}{n}$

$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!},\qquad \cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$

$\int_a^b \sqrt{1 + (dy/dx)^2} \; dx \, ,\qquad \int_a^b 2\pi r\sqrt{1 + (dy/dx)^2} \; dx.$

$\int_a^b y(t)x'(t) \; dt \, ,\qquad \int_a^b \sqrt{x'(t)^2 + y'(t)^2} \; dt \, ,\; \int_a^b 2\pi r\sqrt{x'(t)^2 + y'(t)^2} \; dt \, ,$

$\frac{1}{2}\int_a^b r^2 \; d\theta \, ,\; \int_a^b \sqrt{r(\theta)^2 + r'(\theta)^2} \; d\theta.$

1. Evaluate the following integrals.

(10) a) $\int x^3 \ln(x)\, dx$

(12) b) $\int \frac{x^2\, dx}{\sqrt{x^2 - 1}}$

2. Evaluate the integrals.

(10) a) $\displaystyle\int \frac{e^x}{\sqrt{4 - e^{2x}}}\, dx$

(12) b) $\displaystyle\int \frac{x^2 - 4}{x^3 + x}\, dx$

(10) 3. Evaluate the improper integral or show that it diverges

$\int_0^\infty xe^{-2x^2}\, dx$

4. Let R be the region trapped between $y = 1$ and $y = \cos x$ with $0 \leq x \leq \frac{\pi}{2}$.

(6) a) Find the area of the region R.

(10) b) Find $\overline{x}$, the x coordinate of the centroid of R. (Do not calculate $\overline{y}$.)

(8) 5. Evaluate the series.

$$\sum_{n=1}^{\infty} \sin\left(\frac{\pi}{2}n\right) 2^{-n} =$$

6. Let $S = \sum_{n=1}^{\infty} \frac{(-1)^n}{n!}$.

(4) a) Explain why the series converges.

(4) b) How many terms are required to approximate S with an error less than .01?

(2) c) Evaluate S by using an appropriate series on the cover page.

(12) 7. Solve the initial value problem, $\dfrac{dy}{dt} = \dfrac{\cos^2(y)}{e^{2t}}$, $y(0) = 0$. Express your final answer in the form $y = f(t)$.

(12) 8. Find the interval of convergence of the power series $\displaystyle\sum_{n=1}^{\infty} \frac{(x+2)^n}{\sqrt{n}\, 3^n}$. (Make clear the status of any end points.)

9. Determine whether the following series converge or diverge. State clearly which test you are using and implement the test as clearly as you can. The answer for each problem is worth 2 points and the work you show 4 points.

(6) a) $\displaystyle\sum_{n=3}^{\infty} \frac{1+\frac{1}{n}}{7\cos\frac{1}{n}}$

(6) b) $\displaystyle\sum_{n=2}^{\infty} \frac{n^2+5}{n^{5/2}+n}$

(6) c) $\displaystyle\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$

(12) 10. Find the second degree Taylor polynomial $T_2(x)$ for the function $f(x) = \sqrt{x}$ centered at $x = 4$.

(4) 11. a) Use an appropriate series from the cover sheet to find the Maclaurin series for $\dfrac{1}{1+x^2}$

(4) b) By integrating your expansion in part a) obtain the Maclaurin series for $\tan^{-1} x$.

(2) c) Use part (b) to evaluate the sum $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \cdots =$

(8) 12. Use the series $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2} + \cdots$, and $\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^n}{n} = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \cdots$, to find the terms up to x^4 for the Maclaurin series of $e^{2x}\ln(1+x^2)$.

(6) 13. a) Sketch the graph of the polar equation $r = 6\cos(\theta)$, $0 \le \theta \le \pi$.

(6) b) Convert the polar equation in part a) to a rectangular equation in x and y , and state what familiar shape it is?

14. Consider the curve with parametric equations $x = 4 - \sin(2t)$, $y = 5 + \cos(2t)$.

(6) a) Find the slope of the curve at a general value of t.

(4) b) Find the equation of the tangent line to the curve at $t = \pi/2$.

(8) c) Find the length of the curve for $0 \leq t \leq \pi$.

(10) 15. Calculate the area bounded by one petal of the rose $r = 2\cos(4\theta)$.