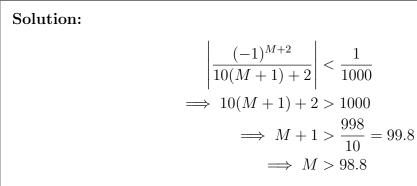
1. (10 points) Evaluate the series $\sum_{n=2}^{\infty} \left(\frac{4}{5}\right)^n$

Solution: This is a geometric series. It converges to the value

$$\frac{(\frac{4}{5})^2}{1-\frac{4}{5}} = \frac{16}{25} \cdot 5 = \boxed{\frac{16}{5}}$$

2. (10 points) Find the minimum M that guarantees that

$$\left|\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{10n+2} - \sum_{n=1}^{M} \frac{(-1)^{n+1}}{10n+2} - \right| < .001.$$



Therefore M = 99 suffices.

3. (10 points) Determine the radius of convergence and interval of convergence, but don't check the endpoints $\sum_{n=1}^{\infty} \frac{2^n}{n} (4x-8)^n$.

Solution:

$$\lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \to \infty} \left| \frac{\frac{2^{n+1}}{n+1} (4x-8)^{n+1}}{\frac{2^n}{n} (4x-8)^n} \right|$$

$$= \lim_{n \to \infty} \left| 2(4x-8) \frac{n}{n+1} \right|$$

$$= 2|4x-8| < 1$$

$$\implies 8|x-2| < 1 \implies |x-2| < \frac{1}{8}$$
The radius of convergence is $\boxed{R = \frac{1}{8}}$
The interval of convergence is $\boxed{\left(\frac{15}{8}, \frac{17}{8}\right)}$

4. (10 points) Show whether the series converges absolutely or conditionally or diverges. Name all tests used.

$$\sum_{n=6}^{\infty} (-1)^n \frac{4 + \sqrt{n}}{n^3 - 5n^2}$$

Solution: Using Limit Comparison Test with $\sum \frac{1}{n^{5/2}}$ (which converges by *p*-series test with $p = \frac{5}{2} > 1$):

$$\lim_{n \to \infty} \left| \frac{4 + \sqrt{n}}{n^3 - 5n} \cdot n^{5/2} \right| = \lim_{n \to \infty} \left| \frac{n^3 + 4n^{5/2}}{n^3 - 5n} \right| = 1$$

By Limit Comparison Test, the given series converges absolutely

5. (10 points) Find the Taylor Series for $f(x) = e^{-x}$ about x = -4.

Solution:

$$f(x) = e^{-x} \quad f(-4) = e^{4}$$

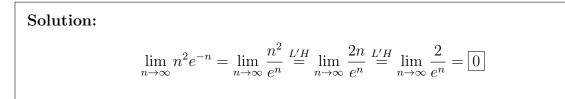
$$f'(x) = -e^{-x} \quad f'(-4) = -e^{4}$$

$$f''(x) = e^{-x} \quad f''(-4) = e^{4}$$

The Taylor series is thus

$$e^{4} - e^{4}(x+4) + \frac{e^{4}}{2!}(x+4)^{2} - \frac{e^{4}}{3!}(x+4)^{3} \dots = \sum_{n=0}^{\infty} \frac{(-1)^{n}e^{4}}{n!}(x+4)^{n}$$

6. (10 points) Find the limit of the sequence or state that it diverges. $\lim_{n \to \infty} n^2 e^{-n}$



Determine whether the following series converge conditionally, converge absolutely, or diverge. Name all tests used.

7. (10 points)
$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^{5/3}}$$

Solution: $\sum \frac{1}{n^{5/3}}$ converges by *p*-series test $(p = \frac{5}{3} > 1)$, so the given series converges absolutely

8. (10 points) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$.

Solution: The absolute value version of the series $\sum \frac{1}{\sqrt{n}}$ diverges by *p*-series test $(p = \frac{1}{2} < 1)$. Trying the Alternating Series Test: $\lim_{n \to \infty} \frac{1}{\sqrt{n}} = 0 \quad \text{and} \quad \frac{1}{\sqrt{n+1}} < \frac{1}{\sqrt{n}}$ so $\sum \frac{(-1)^n}{\sqrt{n}}$ converges by the Alternating Series Test. Thus the given series converges conditionally 9. (10 points) Find a power series representation for the following function and determine its interval of convergence.

$$f(x) = \frac{x}{5-x}$$

Solution: One answer is: $\begin{aligned}
\frac{x}{5-x} &= \frac{x-5+5}{5-x} = -1 + \frac{5}{5-x} = -1 + \frac{5}{1-(x-4)} \\
&= \boxed{-1+5\sum_{n=0}^{\infty}(x-4)^n} \quad (|x-4| < 1)
\end{aligned}$ which has interval of convergence $\boxed{(3,5)}$ Another answer is: $\begin{aligned}
\frac{x}{5-x} &= x \cdot \frac{1}{5-x} = \frac{x}{5} \cdot \frac{1}{1-\frac{x}{5}} = \frac{x}{5} \sum_{n=0}^{\infty} \left(\frac{x}{5}\right)^n \quad \left(\left|\frac{x}{5}\right| < 1 \implies |x| < 5\right) \\
&= \boxed{\sum_{n=0}^{\infty} \left(\frac{x}{5}\right)^{n+1}} \quad (|x| < 5)
\end{aligned}$ which has interval of convergence $\boxed{(-5,5)}$

10. (10 points) Determine whether the following series converge conditionally, converge absolutely, or diverge. Name all tests used. $\sum_{n=1}^{\infty} \left(\frac{3n+1}{4-2n}\right)^n$

Solution: Using root test: $\lim_{n \to \infty} \sqrt[n]{\left| \left(\frac{3n+1}{4-2n} \right)^n \right|} = \lim_{n \to \infty} \left| \frac{3n+1}{4-2n} \right| = \frac{3}{2} > 1$ Thus by root test, the series diverges 11. (10 points) Solve the differential equation for initial conditions y(1) = 4(Not required to solve for y). $\frac{dy}{dx} = x^2y - 3x^2$

Solution:

$$= x^{2}(y-3)$$

$$\int \frac{1}{y-3} dy = \int x^{2} dx$$

$$\ln|y-3| = \frac{x^{3}}{3} + C_{1}$$

$$|y-3| = e^{\frac{x^{3}}{3} + C_{1}} = e^{\frac{x^{3}}{3}}e^{C_{1}} = C_{2}e^{\frac{x^{3}}{3}}$$

$$y-3 = C_{3}e^{\frac{x^{3}}{3}}$$

$$y(x) = Ce^{\frac{x^{3}}{3}} + 3$$

Plugging in the initial condition:

$$4 = Ce^{\frac{1}{3}} + 3 \implies 1 = Ce^{\frac{1}{3}} \implies C = e^{-\frac{1}{3}}$$

The solution is thus

$$y(x) = e^{-\frac{1}{3}}e^{\frac{x^3}{3}} + 3$$