

NAME _____

Rec. Instructor: _____

Signature _____

Rec. Time _____

CALCULUS II - FINAL EXAM Part 1

August 1, 2019

Show all work for full credit. No books, notes or calculators are permitted. The point value of each problem is given in the left-hand margin. You have 1 hour and 15 minutes.

Problem	Points	Points Possible	Problem	Points	Points Possible
1		10	6		10
2		10	7		10
3		10	8		10
4		10	9		10
5		10	Total Score		90

$$\tfrac{d}{dx}\tan x=\sec^2x\qquad\qquad \tfrac{d}{dx}\sec x=\sec x\tan x\qquad\qquad \tfrac{d}{dx}b^x=b^x\ln(b)$$

$$\tfrac{d}{dx}\sin^{-1}(x)=\tfrac{1}{\sqrt{1-x^2}}\qquad\qquad \tfrac{d}{dx}\tan^{-1}(x)=\tfrac{1}{1+x^2}\qquad\qquad \tfrac{d}{dx}\sec^{-1}(x)=\tfrac{1}{|x|\sqrt{x^2-1}}$$

$$\int \tan x ~dx = -\ln |\cos x| + C \qquad\qquad \int \sec x ~dx = \ln |\sec x + \tan x| + C$$

$$\int \frac{dx}{\sqrt{a^2-x^2}}=\sin^{-1}(\frac{x}{a})+C\qquad\qquad \int \frac{dx}{a^2+x^2}=\frac{1}{a}\tan^{-1}(\frac{x}{a})+C$$

$$\int \sin^n x ~dx=-\frac{\sin^{n-1}x\cos x}{n}+\frac{n-1}{n}\int \sin^{n-2}x ~dx,$$

$$\int \cos^n x ~dx=\frac{\cos^{n-1}x\sin x}{n}+\frac{n-1}{n}\int \cos^{n-2}x ~dx,$$

$$\int \tan^n x ~dx=\frac{\tan^{n-1}x}{n-1}-\int \tan^{n-2}x ~dx\,,$$

$$\int \sec^n x ~dx=\frac{\sec^{n-2}x\tan x}{n-1}+\frac{n-2}{n-1}\int \sec^{n-2}x ~dx$$

$$M_x = \tfrac{1}{2} \int_a^b f(x)^2 - g(x)^2 ~dx\,, \quad M_y = \int_a^b x(f(x) - g(x)) ~dx\,.$$

$$|R_n(x)|\leq \tfrac{K}{(n+1)!}|x-a|^{n+1}\,,\text{ with }\, K=\max_{a\leq c\leq x}|f^{(n+1)}(c)|\,.$$

$$\tfrac{1}{1-x}=\sum\nolimits_{n=0}^\infty x^n\,,\quad\; e^x=\sum\nolimits_{n=0}^\infty \tfrac{x^n}{n!},\quad\; \ln(1+x)=\sum\nolimits_{n=1}^\infty \tfrac{(-1)^{n+1}x^n}{n}$$

$$\sin x=\sum\nolimits_{n=0}^\infty \tfrac{(-1)^nx^{2n+1}}{(2n+1)!},\quad\; \cos x=\sum\nolimits_{n=0}^\infty \tfrac{(-1)^nx^{2n}}{(2n)!}$$

$$\int_a^b \sqrt{1+(dy/dx)^2}~dx\,,\quad \int_a^b 2\pi r\sqrt{1+(dy/dx)^2}~dx.$$

$$\int_a^b y(t)x'(t)~dt\,,\quad \int_a^b \sqrt{x'(t)^2+y'(t)^2}~dt\,,~\int_a^b 2\pi r\sqrt{x'(t)^2+y'(t)^2}~dt\,,$$

$$\tfrac{1}{2}\int_a^b r^2~d\theta\,,~\int_a^b \sqrt{r(\theta)^2+r'(\theta)^2}~d\theta\,,~~\int_a^b \sqrt{r^2+\left(\tfrac{dr}{d\theta}\right)^2}d\theta$$

$$(10) \quad \text{1. } \int \frac{x^2 \, dx}{\sqrt{1 - x^2}}$$

$$(10) \quad \text{2. } \int \frac{7 - 3x}{x^2 + 2x - 3} dx$$

- $$(10)$$
3. Find the third degree Taylor polynomial $T_3(x)$ for the function $f(x) = \sin(x)$ centered at $x = 4$.

(10)

4. Does the series conditionally converge, absolutely converge, or diverge? Name all tests used

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n!}$$

(10)

5. Find the arc length from of $y = 2(1 + x)^{\frac{3}{2}}$ from $x = 0$ to 1

(10)

6. Solve the initial value problem, $\frac{dy}{dt} = \frac{e^{-3t}}{\sin(y)}$, $y(0) = 0$. It's not necessary to solve for y .

(10)

7. Does the series conditionally converge, absolutely converge, or diverge? Name each test used

$$\sum_{n=3}^{\infty} \frac{e^n}{\frac{1}{n} - 9}$$

(10)

8. (a) Use an appropriate series from the cover sheet to find the Maclaurin series for

$$\frac{1}{1 + 3x}$$

- (b) Differentiating (a) gives $\frac{-3}{(1+3x)^2}$. Find its Maclaurin series.

(10)

9. Determine the center of mass for $y = x^3$ and $y = x$ from $x = 0$ to 1