1. Evaluate the following integrals.

(a) (10 points) $\displaystyle\int \frac{\sin\left(\sqrt{x}\right)}{\sqrt{x}}\,\mathrm{d}x$

Solution: Let $u = \sqrt{x}$, so $\mathrm{d}u = \dfrac{1}{2\sqrt{x}}\,\mathrm{d}x$. Then

$$\begin{aligned}\int \frac{\sin\left(\sqrt{x}\right)}{\sqrt{x}}\,\mathrm{d}x &= 2\int \sin u\,\mathrm{d}u \\ &= -2\cos u \\ &= \boxed{-2\cos\left(\sqrt{x}\right) + C}\end{aligned}$$

(b) (10 points) $\displaystyle\int \frac{x}{\sqrt{x+1}}\,\mathrm{d}x$

Solution: Letting $u = x + 1$, so $\mathrm{d}u = \mathrm{d}x$ and $u - 1 = x$ Then

$$\begin{aligned}\int \frac{x}{\sqrt{x+1}}\,\mathrm{d}x &= \int \frac{u-1}{\sqrt{u}}\,\mathrm{d}u \\ &= \int u^{1/2} - u^{-1/2} \\ &= \frac{2}{3}u^{3/2} - 2u^{1/2} \\ &= \boxed{\frac{2}{3}(x+1)^{3/2} - 2(x+1)^{1/2} + C}\end{aligned}$$

2. Evaluate the following integrals.

(a) (10 points) $\int \sin^{-1} x \, \mathrm{d}x$, where $\sin^{-1} x$ is the inverse sine function.

Solution: IBP:

	D	I
$+$	$\sin^{-1} x$	1
$-$	$\dfrac{1}{\sqrt{1-x^2}}$	x

so

$$\begin{aligned}\int \sin^{-1} x \, \mathrm{d}x &= x \sin^{-1} x - \int \frac{x}{\sqrt{1-x^2}} \qquad (u = 1 - x^2; \mathrm{d}u = -2x \, \mathrm{d}x) \\ &= x \sin^{-1} x + \frac{1}{2} \int \frac{1}{\sqrt{u}} \, \mathrm{d}u \\ &= x \sin^{-1} x + u^{1/2} \\ &= \boxed{x \sin^{-1} x + \sqrt{1-x^2} + C}\end{aligned}$$

(b) (10 points) $\int x^7 \ln x \, \mathrm{d}x$,

Solution: IBP:

	D	I
$+$	$\ln x$	x^7
$-$	$\dfrac{1}{x}$	$\dfrac{x^8}{8}$

so

$$\begin{aligned}\int x^7 \ln x \, \mathrm{d}x &= \frac{x^8}{8} \ln x - \int \frac{1}{8} x^7 \, \mathrm{d}x \\ &= \boxed{\frac{x^8}{8} \ln x - \frac{1}{64} x^8 + C}\end{aligned}$$

3. Evaluate the following integrals.

   (a) (10 points) $\displaystyle\int_0^{\pi/4} \sin(4x)\sin(2x)\,\mathrm{d}x$. Express your final answer as a reduced fraction $\dfrac{a}{b}$, with no trig functions.

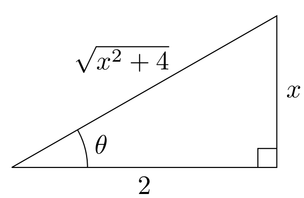
   Solution: Starting with the sum-to-product formula on the cover page,

$$\begin{aligned}\int_0^{\pi/4} \sin(4x)\sin(2x)\,\mathrm{d}x &= \frac{1}{2}\int_0^{\pi/4} \cos(2x) - \cos(6x)\,\mathrm{d}x \\ &= \frac{1}{2}\left[\frac{1}{2}\sin(2x) - \frac{1}{6}\sin(6x)\right]_0^{\pi/4} \\ &= \frac{1}{2}\left[\frac{1}{2}\sin\left(\frac{\pi}{2}\right) - \frac{1}{6}\sin\left(\frac{3\pi}{2}\right)\right] \\ &= \frac{1}{2}\left[\frac{1}{2}+\frac{1}{6}\right] \\ &= \boxed{\frac{1}{3}}\end{aligned}$$

   (b) (10 points) $\displaystyle\int \frac{\mathrm{d}x}{\sqrt{4+x^2}}$

   Solution: Trig sub: $x = 2\tan\theta$, so $\mathrm{d}x = 2\sec^2\theta\,\mathrm{d}\theta$,

$$\begin{aligned}\int \frac{\mathrm{d}x}{\sqrt{4+x^2}} &= \int \frac{2\sec^2\theta\,\mathrm{d}\theta}{\sqrt{4+4\tan^2\theta}} \\ &= \int \frac{2\sec^2\theta\,\mathrm{d}\theta}{\sqrt{4(1+\tan^2\theta)}} \\ &= \int \frac{2\sec^2\theta\,\mathrm{d}\theta}{2\sqrt{\sec^2\theta}} \\ &= \int \sec\theta\,\mathrm{d}\theta \\ &= \ln|\sec\theta+\tan\theta| \\ &= \boxed{\ln\left|\frac{\sqrt{x^2+4}}{2}+\frac{x}{2}\right|+C}\end{aligned}$$



4. Evaluate the following integrals.

(a) (10 points) $\int \sin^4(x)\cos^7(x)\,\mathrm{d}x$

Solution:

$$\begin{aligned}
\int \sin^4(x)\cos^7(x)\,\mathrm{d}x &= \int \sin^4(x)\underbrace{\cos^6(x)}_{(\cos^2(x))^3}\cos x\,\mathrm{d}x \\
&= \int \sin^4(x)\underbrace{(1-\sin^2(x))^3}_{\substack{\text{rewritten with}\\ \cos^2=1-\sin^2}}\cos x\,\mathrm{d}x \\
&= \int u^4(1-u^2)^3\,\mathrm{d}u \\
&= \int u^4(1-3u^2+3u^4-u^6)\,\mathrm{d}u \\
&= \int u^4-3u^6+3u^8-u^{10}\,\mathrm{d}u \\
&= \frac{u^5}{5}-\frac{3}{7}u^7+\frac{1}{3}u^9-\frac{1}{11}u^{11} \\
&= \boxed{\frac{1}{5}\sin^5(x)-\frac{3}{7}\sin^7(x)+\frac{1}{3}\sin^9(x)-\frac{1}{11}\sin^{11}(x)+C}
\end{aligned}$$

(b) (10 points) $\int \sec^4(x)\,\mathrm{d}x$

Solution: Using reduction formula on cover page:

$$\begin{aligned}
\int \sec^4(x)\,\mathrm{d}x &= \frac{\sec^2(x)\tan(x)}{3}+\frac{2}{3}\int \sec^2(x)\,\mathrm{d}x \\
&= \boxed{\frac{\sec^2(x)\tan(x)}{3}+\frac{2}{3}\tan x+C}
\end{aligned}$$

5. (10 points) An object moves along a straight line with velocity function $v(t) = t\sin t$, in meters per second. Determine its change in position over the time interval $t = 0$ to $t = \pi$ seconds. (Evaluate any trig function in your answer.)

Solution: The object's displacement is given by the integral $\int_0^{\pi} t\sin t\,\mathrm{d}t$. Evaluating this uses IBP:

	D	I
$+$	t	$\sin t$
$-$	1	$-\cos t$
$+$	0	$-\sin t$

Thus

$$\int_0^{\pi} t\sin t\,\mathrm{d}t = -t\cos t + \sin t\Big|_0^{\pi} = \pi - 0 = \boxed{\pi \text{ meters}}$$

6. (10 points) Find a function $f(t)$ such that $f'(t) = \dfrac{\ln t}{t} - \cos(2\pi t)$.

Solution:

$$\int \left(\frac{\ln t}{t} - \cos(2\pi t)\right)\mathrm{d}t = \boxed{\frac{1}{2}(\ln t)^2 - \frac{1}{2\pi}\sin(2\pi t) + C}$$

(The first term is integrated with $u = \ln t$. The second term can be done with $u = 2\pi t$, though it is also a guess-and-fudge type of u-sub.)