NAME _____ Rec. Instructor: _____

Signature _____ Rec. Time _____

CALCULUS II - EXAM 2
March 3, 2020

Show all work for full credit. No books, notes or calculators are permitted. The point value of each problem is given in the left-hand margin. You have 75 minutes.

Problem	Points	Possible	Problem	Points	Possible
1a		10	4b		6
1b		12	4c		4
2a		8	5a		8
2b		8	5b		10
3		8	6		10
4a		6	7		10
			Total Score		100

You are free to use the following formulas on any of the problems.

$$\int \tan x \, dx = -\ln|\cos x| + C, \qquad \int \sec x \, dx = \ln|\sec x + \tan x| + C$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C, \qquad \int \sqrt{x^2 + a^2} \, dx = \tfrac{1}{2}\left(x\sqrt{x^2 + a^2} + a^2 \ln|x + \sqrt{x^2 + a^2}|\right) + C$$

$$\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C, \qquad \int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right) + C$$

$T_n = \frac{\Delta x}{2}(f(x_0) + 2f(x_1) + \cdots + 2f(x_{n-1}) + f(x_n))$,

$S_n = \frac{\Delta x}{3}\left(f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots + 4f(x_{n-1}) + f(x_n)\right)$.

Work = Force $\times$ Distance; Units of work: ft-lbs, newton-meters = joules;

Hooke's Law for springs: $F = kx$, where x is the distance stretched from rest position.

$M_x = \frac{1}{2}\int_a^b f(x)^2 - g(x)^2 \, dx, \quad M_y = \int_a^b x(f(x) - g(x)) \, dx.$

1. Evaluate the following integrals.

(10) a) $\displaystyle\int \frac{x^3 + 2x + 1}{x^2 + 4}\, dx$

(12) b) $\displaystyle\int \frac{3x + 5}{(x^2 + 2x + 1)(x + 2)}\, dx$

2. Approximate the definite integral $\int_{-4}^{4} \sqrt{16 - x^2}\, dx$ using

(8) a) The Midpoint rule for M_4. (Do not simplify the arithmetic.)

(8) b) Simpson's rule for S_4. (Do not simplify the arithmetic.)

(8) **3.** A spring requires a force of 4 newtons to stretch it 2 meters beyond its rest length. How much work is required to stretch the spring from 2 meters to 4 meters beyond its rest length?

4. Evaluate the following improper integrals, or state that they do not exist. Use proper limit notation.

(6) a) $\displaystyle\int_2^5 \frac{dx}{\sqrt{x-2}}$

(6) b) $\displaystyle\int_3^\infty \frac{dx}{(x-2)^3}$

(4) c) $\displaystyle\int_{-2}^2 \frac{dx}{x^2}$

(8) **5.** a) Find the arc length of the curve $y = \sin x$, $0 \leq x \leq \frac{\pi}{2}$. Just set up the integral. **Do not evaluate.**

(10) b) Find the surface area of the surface generated by rotating the curve in part a) around the x-axis. **Evaluate the integral.** Make use of an appropriate integral formula on the cover page.

(10) **6.** How much work is done by winding up a hanging cable of length 50 feet and weight density 2 lb/ft.

(10) **7.** Find the centroid $(\overline{x}, \overline{y})$ of the region bounded by the semicircle $y = \sqrt{4 - x^2}$, $-2 \leq x \leq 2$ and the x-axis. (You may use the area formula for a circle, and symmetry to determine one of the values $\overline{x}, \overline{y}$.)