NAME ______________________ Rec. Instructor: ______________

Signature ______________________ Rec. Time __________

CALCULUS II - EXAM 3
April 14, 2020

Show all work for full credit. No books, notes or calculators are permitted. The point value of each problem is given in the left-hand margin. You have 75 minutes.

Problem	Points	Possible	Problem	Points	Possible
1a		8	5		7
1b		8	6a		7
2		10	6b		7
3a		6	6c		7
3b		6	7		6
4a		6	8a		7
4b		8	8b		7
			Total Score		100

You are free to use the following formulas on any of the problems.

$\cosh(x) = \frac{1}{2}(e^x + e^{-x})$, $\sinh(x) = \frac{1}{2}(e^x - e^{-x})$,

$\cosh^2 x - \sinh^2 x = 1$, $\cosh^2 x = \frac{1}{2}(1 + \cosh(2x))$, $\sinh(2x) = 2\sinh(x)\cosh(x)$

$\frac{d}{dx}\sinh(x) = \cosh(x)$, $\frac{d}{dx}\cosh(x) = \sinh(x)$

$\frac{d}{dx}\tanh(x) = \text{sech}^2 x$, $\frac{d}{dx}\text{sech}(x) = -\text{sech}(x)\tanh(x)$.

$\frac{d}{dx}\sinh^{-1}(x) = \frac{1}{\sqrt{1+x^2}}$, $\frac{d}{dx}\cosh^{-1}(x) = \frac{1}{\sqrt{x^2-1}}$,

$\frac{d}{dx}\tanh^{-1}(x) = \frac{1}{1-x^2}$, $\frac{d}{dx}\text{sech}^{-1}(x) = \frac{-1}{x\sqrt{1-x^2}}$

1. Evaluate the following.

(8) a) $\dfrac{d}{dx}\, x^3 \tanh^{-1}(e^{2x})$, where $\tanh^{-1}$ is the inverse tanh function.

(8) b) $\displaystyle\int (2 + \cosh x)(1 + \sinh x)\, dx$,

2. Consider the differential equation

$$\frac{dy}{dx} = \frac{xy}{\ln y}$$

(8) a) Find the general solution. Solve for y explicitly.

(2) b) Find the solution satisfying the initial condition $y(0) = e$ where e is the natural log base.

3. Find the limit of the sequence or state that it diverges.

(6) a) $\lim_{n\to\infty} e^{-2n}(n^2+1)$

(6) b) $\lim_{n\to\infty} \frac{\sin(5/n)}{\sin(3/n)}$

4. Evaluate the series.

(6) $$\sum_{n=0}^{\infty} \frac{5}{2} \cdot \left(\frac{2}{5}\right)^n = \frac{5}{2} + 1 + \frac{2}{5} + \frac{4}{25} + \frac{8}{125} \cdots .$$

(8) b. $$\sum_{n=2}^{\infty} \frac{2}{n^2 - n}$$

(7) **5.** Use the limit comparison test to determine whether the following series converges or diverges. Show all work to justify your answer.

$$\sum_{n=1}^{\infty} \frac{n^2 - n}{3n^{5/2} + 217}$$

6. Determine whether the following series converge or diverge. Show all work to justify your answers.

(7) a. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$

(7) b. $\sum_{n=2}^{\infty} \frac{1}{n(\ln(n))^2}$

(7) c. $\sum_{n=1}^{\infty} (-1)^{n+1} \cos(n\pi/2)$

(6) **7.** The infinite series $S = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{3n-1}$ is estimated using the M-th partial sum S_M. Find the minimum M that guarantees that $|S - S_M| < .01$.

8. Determine whether the following series converge conditionally, converge absolutely, or diverge. Justify your answer.

(7) a. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$

(7) b. $\displaystyle\sum_{n=1}^{\infty} \frac{\sin(2n)}{n^2 + n}$