NAME ____________________ Rec. Instructor: ____________

Signature ____________________ Rec. Time ________

CALCULUS II - FINAL EXAM
May 13, 2020, 6:30 to 9:00 p.m.

Show all work for full credit. No books, notes or calculators are permitted. The point value of each problem is given in the left-hand margin. You have 2 hours and 30 minutes. **After 8:40, only uploading/submitting is permitted.**

Problem	Points	Points Possible	Problem	Points	Points Possible
1		10	11		12
2		12	12		6
3		10	13		12
4		12	14		8
5		10	15		15
6		8	16		12
7		16	17		8
8		8	18		11
9		12	19		6
10		12	Total Score		200

$\frac{d}{dx}\tan x = \sec^2 x$ $\qquad$ $\frac{d}{dx}\sec x = \sec x \tan x$ $\qquad$ $\frac{d}{dx}b^x = b^x \ln(b)$

$\frac{d}{dx}\sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}}$ $\qquad$ $\frac{d}{dx}\tan^{-1}(x) = \frac{1}{1+x^2}$ $\qquad$ $\frac{d}{dx}\sec^{-1}(x) = \frac{1}{|x|\sqrt{x^2-1}}$

$$\int \tan x \; dx = -\ln|\cos x| + C \qquad \int \sec x \; dx = \ln|\sec x + \tan x| + C$$

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C \qquad \int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \sin^n x \; dx = -\frac{\sin^{n-1} x \cos x}{n} + \frac{n-1}{n}\int \sin^{n-2} x \; dx,$$

$$\int \cos^n x \; dx = \frac{\cos^{n-1} x \sin x}{n} + \frac{n-1}{n}\int \cos^{n-2} x \; dx,$$

$$\int \tan^n x \; dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x \; dx\,,$$

$$\int \sec^n x \; dx = \frac{\sec^{n-2} x \tan x}{n-1} + \frac{n-2}{n-1}\int \sec^{n-2} x \; dx$$

$M_x = \frac{1}{2}\int_a^b f(x)^2 - g(x)^2 \; dx\,, \qquad M_y = \int_a^b x(f(x)-g(x)) \; dx\,.$

$|R_n(x)| \leq \frac{K}{(n+1)!}|x-a|^{n+1}$, with $K = \max_{a\leq c\leq x}|f^{(n+1)}(c)|\,.$

$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n\,,$ $\qquad$ $(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots$

$e^x = \sum_{n=0}^{\infty}\frac{x^n}{n!}\,,$ $\qquad$ $\ln(1+x) = \sum_{n=1}^{\infty}\frac{(-1)^{n+1}x^n}{n}$

$\sin x = \sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+1}}{(2n+1)!}\,,$ $\qquad$ $\cos x = \sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}}{(2n)!}$

$\int_a^b \sqrt{1+(dy/dx)^2} \; dx\,, \qquad \int_a^b 2\pi r\sqrt{1+(dy/dx)^2} \; dx.$

$\int_a^b \sqrt{x'(t)^2+y'(t)^2} \; dt\,, \qquad \frac{dy}{dx} = \frac{dy/dt}{dx/dt}\,.$

$\frac{1}{2}\int_a^b r^2 \; d\theta\,, \qquad \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}\,.$

(10) 1. Evaluate the following integral.

$\int x \tan^{-1}(x)\, dx$, where $\tan^{-1}(x)$ is the inverse tangent function.

(12) 2. Evaluate the following integral. $\int \frac{x^2\ dx}{\sqrt{1-x^2}}$

(10) 3. Evaluate the integral.

$$\int \frac{e^{2x}}{5+e^{2x}}\ dx$$

(12) 4. Evaluate the integral. $\displaystyle\int \frac{3x^2+8}{x^3+4x}\ dx$

(10) 5. Evaluate the improper integral or show that it diverges. Use limit notation.

$$\int_1^5 \frac{dx}{\sqrt{x-1}}$$

(8) 6. Explain why the following series converges, and then evaluate it.

$$\sum_{n=1}^{\infty} \frac{2}{\pi}\left(-\frac{\pi}{4}\right)^n$$

7. Let R be the region trapped between $y = x$ and $y = x^2$, with $0 \leq x \leq 1$.

(6) a) Find the area of the region R.

(10) b) Find $\overline{y}$, the y coordinate of the centroid of R. (Do not calculate $\overline{x}$.)

8. Let $S = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n+1}$.

(4) a) Explain why the series converges.

(4) b) How many terms are required to approximate S with an error less than .01?

(12) 9. Find the interval of convergence of the power series $\displaystyle\sum_{n=1}^{\infty} \frac{(x-2)^n}{n\ 4^n}$. (Make clear the status of any end points.)

(12) 10. Solve the initial value problem, $(1+t^2)\frac{dy}{dt} = 2te^y$, $y(0) = 1$. Express your final answer in the form $y = f(t)$.

11. Determine whether the following series converge or diverge. State clearly which test you are using and implement the test as clearly as you can. The answer is worth 2 points and the work you show 4 points.

(6) a) $\displaystyle\sum_{n=3}^{\infty} \frac{\sqrt{n}}{n^2+1}$

(6) b) $\displaystyle\sum_{n=2}^{\infty} \frac{\cos(1/n)}{2+\sin(1/n)}$

(6) 12. Determine whether the following series converges or diverges. State clearly which test you are using and implement the test as clearly as you can. The answer is worth 2 points and the work you show 4 points.

$$\sum_{n=1}^{\infty} \left(\frac{2n^2-3}{n^2+7n}\right)^n$$

(12) 13. Find the third degree Taylor polynomial $T_3(x)$ for the function $f(x) = \cos x$ centered at $x = \pi/2$.

(8) 14. Use the series expansions for e^x and $(1+x)^n$ on the cover page to find the terms up to x^4 for the Maclaurin series of $e^{x^2}\sqrt{1+x^2}$

(5) 15. a) Use an appropriate series from the cover sheet to find the Maclaurin series for

$$\frac{1}{1+x^5}$$

(3) b) Use your answer to part a) to find the Maclaurin series for

$$\frac{x^3}{1+x^5}$$

(7) c) Use your answer to part a) to evaluate the following integral, expressing your final answer as an infinite series.

$$\int_0^1 \frac{dx}{1+x^5}$$

16. Consider the curve with parametric equations $x = t^2 + 1$, $y = 2t^3$, $t \geq 0$.

(4) a) Find the slope of the curve at a general value t.

(4) b) Find the equation of the tangent line to the curve at $t = 1$.

(4) c) Convert the equation to a rectangular equation in x, y.

(8) 17. Find the length of the curve $x = t^2 + 1$, $y = 2t^3$, $0 \leq t \leq 1$.

(6) 18. a) Convert the polar equation $r = 2\sin\theta + 4\cos\theta$, to a rectangular equation in x, y.

(5) b) Sketch the graph of the polar equation in part a) with $0 \leq \theta \leq \frac{\pi}{2}$.
(Note: $\sqrt{2} \approx 1.4$, $\sqrt{3} \approx 1.7$, $\sqrt{5} \approx 2.2$.)

(6) 19. Find the slope of the polar curve $r = 2\sin\theta + 4\cos\theta$ at $\theta = \pi/2$.