Name: ______________________________

Recitation time: ____________________ **Rec. instructor:** ______________________

MATH 221 - Final
December 15, 2021

- This exam contains 12 pages (including this cover page) and 13 questions.
- No books, calculators, or notes are allowed. You must show all your work to get credit for your answers.
- You have 1 hour and 50 minutes to complete the exam.

Question	Points	Score
1	22	
2	22	
3	8	
4	16	
5	12	
6	12	
7	18	
8	18	
9	14	
10	12	
11	18	
12	16	
13	12	
Total:	200	

$$\frac{d}{dx}\tan x = \sec^2 x \qquad \frac{d}{dx}\sec x = \sec x \tan x \qquad \frac{d}{dx}b^x = b^x \ln b$$

$$\int \tan x\,dx = \ln|\sec x| + C \qquad \int \sec x\,dx = \ln|\sec x + \tan x| + C$$

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C, \quad \int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C, \quad \int \frac{1}{x\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \sin^n(x)\ dx = -\frac{\sin^{n-1}(x)\cos(x)}{n} + \frac{n-1}{n}\int \sin^{n-2}(x)\ dx$$

$$\int \cos^n(x)\ dx = \frac{\cos^{n-1}(x)\sin(x)}{n} + \frac{n-1}{n}\int \cos^{n-2}(x)\ dx$$

$$\int \tan^n(x)\ dx = \frac{\tan^{n-1}(x)}{n-1} - \int \tan^{n-2}(x)\ dx$$

$$\int \sec^n(x)\ dx = \frac{\sec^{n-2}(x)\tan(x)}{n-1} + \frac{n-2}{n-1}\int \sec^{n-2}(x)\ dx$$

$$M_x = \frac{1}{2}\int_a^b f(x)^2 - g(x)^2 dx \qquad M_y = \int_a^b x(f(x) - g(x))dx$$

$$L = \int_a^b \sqrt{1 + (dy/dx)^2}dx \ , \quad SA = \int_a^b 2\pi r\sqrt{1 + (dy/dx)^2}dx$$

$$|R_n(x)| \le \frac{K}{(n+1)!}|x-a|^{n+1}, \quad \text{with} \quad K = \max_{a \le c \le x}|f^{(n+1)}(c)|.$$

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \ , \quad e^x = \sum_{n=0}^{\infty}\frac{x^n}{n!} \ , \quad \ln(1+x) = \sum_{n=1}^{\infty}\frac{(-1)^{n+1}x^n}{n}$$

$$\sin x = \sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+1}}{(2n+1)!} \ , \quad \cos x = \sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}}{(2n)!}$$

$$A = \int_a^b y(t)x'(t)dt \ , \quad L = \int_a^b \sqrt{x'(t)^2 + y'(t)^2}dt$$

$$A = \frac{1}{2}\int_a^b r^2 d\theta \ , \quad L = \int_a^b \sqrt{r(\theta)^2 + r'(\theta)^2}d\theta$$

1. Evaluate the following integrals

   (a) (11 points) $\int 5x \ln(4x)\, dx$

   (b) (11 points) $\int \frac{2x^2 - 1}{x^3 + x^2}\, dx$

2. Evaluate the following integrals

(a) (11 points) $\displaystyle\int \frac{e^x}{1+e^{2x}}\,dx$

(b) (11 points) $\displaystyle\int \tan(x)\sec^4(x)\,dx$

3. (8 points) Set up the integral that computes the area of the surface obtained by rotating the curve $y = \sqrt{1 - x^2}$, $-1 \leq x \leq 1$ around the $x-$axis. **Do not evaluate the integral.**

4. Let R be the region trapped between $y = 1$ and $y = \cos x$, with $0 \leq x \leq \frac{\pi}{2}$.

   (a) (6 points) Find the area of the region R.

   (b) (10 points) Find $\bar{x}$, the x coordinate of the centroid of R. (Do not calculate $\bar{y}$)

5. (12 points) Find the general solution of the differential equation $\dfrac{dy}{dx} = 3x^2y^2$.

6. (12 points) Use the integral test to determine if the series $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\ln^2(n)}$ converges or diverges.

7. Determine if the following series converge or diverge

(a) (9 points) $\displaystyle\sum_{n=1}^{\infty} \frac{7n-5}{n^3+n+1}$

(b) (9 points) $\displaystyle\sum_{n=2}^{\infty} \frac{n}{\ln n}$

8. (a) (9 points) Evaluate the series $\sum_{n=0}^{\infty} \frac{2^n + 4}{3^n}$.

(b) (9 points) Determine whether the series $\sum_{n=0}^{\infty} \frac{(-1)^n}{n!}$ converges absolutely, conditionally or diverges.

9. (14 points) Find the interval of convergence for the power series

$$\sum_{n=1}^{\infty} \frac{(x-1)^n}{n^2 4^n}$$

10. (12 points) Find the degree two Taylor polynomial of $f(x) = \ln(5x)$ centered at $x = 1$.

11. Using the appropriate series from the formula sheet, find the Maclaurin series of:

(a) (9 points) $f(x) = \dfrac{x^3}{1-x^2}$

(b) (9 points) $g(x) = \displaystyle\int \frac{\sin x}{x}\, dx$

12. Consider the curve with parametric equations $x = e^t + 2$, $y = 2e^t + 5$ for $-1 \leq t \leq 1$.

(a) (5 points) Find the slope of the curve at a general value of t.

(b) (5 points) Find the equation of the tangent line to the curve at $t = 0$.

(c) (6 points) Find the length of the curve.

13. (12 points) Calculate the area bounded by one petal of the rose $r = 4\sin 3\theta$.