Name: _____

Recitation time: _____ **Rec. instructor:** _____

MATH 221 - Midterm 1
January 31, 2023

- This exam contains 7 pages (including this cover page) and 7 questions.
- Answer the questions in the spaces provided in this booklet.
- No books, calculators, or notes are allowed. You must show all your work to get credit for your answers.
- You have 1 hour and 15 minutes to complete the exam.

Question:	1	2	3	4	5	6	7	Total
Points:	18	18	18	18	10	8	10	100
Score:								

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C, \quad \int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C, \quad \int \frac{1}{x\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \tan x\,dx = \ln|\sec x| + C \qquad \int \sec x\,dx = \ln|\sec x + \tan x| + C$$

$$\int \sin^n(x)\,dx = -\frac{\sin^{n-1}(x)\cos(x)}{n} + \frac{n-1}{n}\int \sin^{n-2}(x)\,dx$$

$$\int \cos^n(x)\,dx = \frac{\cos^{n-1}(x)\sin(x)}{n} + \frac{n-1}{n}\int \cos^{n-2}(x)\,dx$$

$$\int \tan^n(x)\,dx = \frac{\tan^{n-1}(x)}{n-1} - \int \tan^{n-2}(x)\,dx$$

$$\int \sec^n(x)\,dx = \frac{\sec^{n-2}(x)\tan(x)}{n-1} + \frac{n-2}{n-1}\int \sec^{n-2}(x)\,dx$$

$$\sin^2(x) = \frac{1-\cos(2x)}{2} \qquad \cos^2(x) = \frac{1+\cos(2x)}{2}$$

$$\sin(ax)\sin(bx) = \frac{1}{2}\cos((a-b)x) - \frac{1}{2}\cos((a+b)x)$$

$$\cos(ax)\cos(bx) = \frac{1}{2}\cos((a-b)x) + \frac{1}{2}\cos((a+b)x)$$

$$\sin(ax)\cos(bx) = \frac{1}{2}\sin((a-b)x) + \frac{1}{2}\sin((a+b)x)$$

1. Evaluate the following integrals

(a) (9 points) $\int \frac{1}{x(\ln x)^2}\,dx$

(b) (9 points) $\int x^2\sqrt{x^3+5}\,dx$

2. Evaluate the following integrals.

(a) (9 points) $\int x^5 \ln x \, dx$

(b) (9 points) $\int \sin^{-1}(x) \, dx$, where $\sin^{-1}(x) = \arcsin(x)$.

3. Evaluate the following integrals.

(a) (9 points) $\int \sec^4(x)\tan^4(x)\,dx$

(b) (9 points) $\int \tan^4(x)\,dx$

4. Evaluate the following integrals.

(a) (11 points) $\displaystyle\int \frac{1}{\sqrt{x^2-9}}\,dx$

(b) (7 points) $\displaystyle\int \frac{5}{16+x^2}\,dx$

5. (10 points) Evaluate the following integral

$$\int_0^{\pi/2} \sin^3(x)\cos^2(x)\,dx$$

6. (8 points) Find a function $f(t)$ such that

$$f'(t) = \sin(t)\sin(3t)$$

7. (10 points) Find the area of the region bounded by the curves $y = 0$, $x = 1$ and $y = xe^{-x}$.