Name: _____

Recitation time: _____ **Rec. instructor:** _____

MATH 221 - Midterm 2
February 28, 2023

- This exam contains 6 pages (including this cover page) and 8 questions.
- Answer the questions in the spaces provided in this booklet.
- No books, calculators, or notes are allowed. You must show all your work to get credit for your answers.
- You have 1 hour and 15 minutes to complete the exam.

Question:	1	2	3	4	5	6	7	8	Total
Points:	11	9	9	11	18	11	11	20	100
Score:									

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C, \qquad \int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C,$$
$$\int \frac{1}{x\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \tan x\,dx = \ln|\sec x| + C \qquad \int \sec x\,dx = \ln|\sec x + \tan x| + C$$

$$\sin^2(x) = \frac{1-\cos(2x)}{2} \qquad \cos^2(x) = \frac{1+\cos(2x)}{2}$$

$$\text{Error}(M_n) \le \frac{M(b-a)^3}{24n^2} \quad \text{Error}(T_n) \le \frac{M(b-a)^3}{12n^2} \quad \text{Error}(S_n) \le \frac{M^*(b-a)^5}{180n^4}$$

$M = \max \text{ value } |f^{(2)}(x)| \quad M^* = \max \text{ value } |f^{(4)}(x)|$

$$M_x = \frac{\rho}{2}\int_a^b f(x)^2 - g(x)^2\,dx \qquad M_y = \rho\int_a^b x(f(x)-g(x))\,dx$$

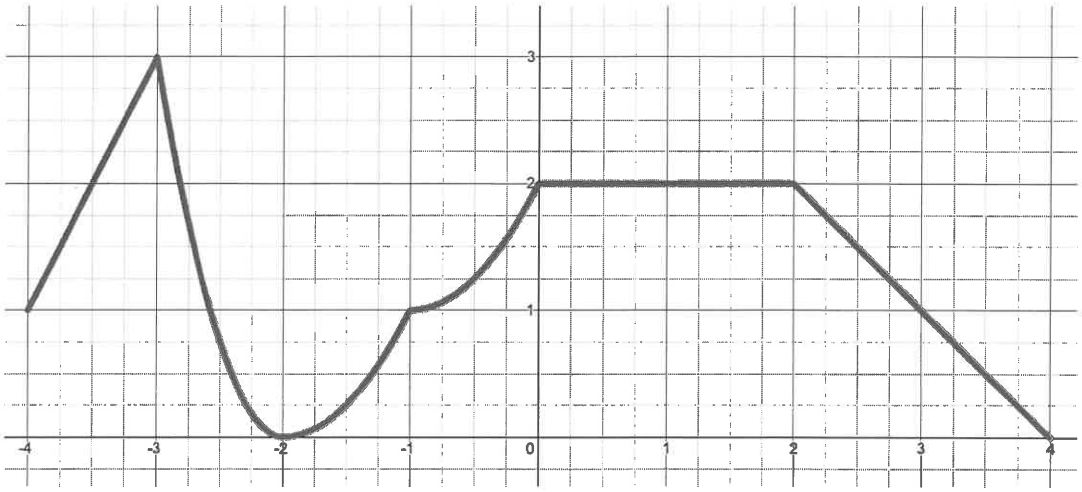
1. (11 points) Evaluate the following integral

$$\int \frac{3x^4 + x^3 + 13x^2 + 4x + 11}{x^2 + 4}\, dx$$

2. (9 points) Give the partial fractions expansion of the rational function $f(x)$ using coefficients $A, B, C, \ldots$, but do NOT solve the values of the coefficients.

$$f(x) = \frac{6x^2 + 3x - 7}{(x^2 - 1)^2(x^2 + 4)^2}$$

3. (9 points) Consider the function $y = f(x)$ graphed below.



Approximate the definite integral $\int_{-4}^{4} f(x)\,dx$ using the midpoint rule M_4.

4. (11 points) How large would n need to guarantee that an estimate of $\int_{0}^{2} x^3\,dx$ is accurate to within 0.1 if we use the trapezoid rule T_n ?

5. Evaluate the following integrals using proper limit notation

(a) (9 points) $\int_0^{\infty} 3xe^{-x^2}\,dx$

(b) (9 points) $\int_0^4 \frac{3}{\sqrt{4-x}}\,dx$

6. (11 points) Find the length of the curve $y = \sqrt{9 - x^2}$, $-3 \leq x \leq 3$.

7. (11 points) Find the surface area of the surface obtained by rotating the curve $y = x^2$, $1 \leq x \leq 2$ around the y-axis.

8. (20 points) Find $\bar{x}$, the x-coordinate of the center of mass (centroid) of the region bounded by $y = e^x$, $x = 0$, $x = 1$ and the x-axis.