Name: _____

Recitation time: _____ **Rec. instructor:** _____

MATH 221 - Midterm 3
April 4, 2023

- This exam contains 7 pages (including this cover page) and 6 questions.
- Answer the questions in the spaces provided in this booklet.
- No books, calculators, or notes are allowed. You must show all your work to get credit for your answers.
- You have 1 hour and 15 minutes to complete the exam.

Question:	1	2	3	4	5	6	Total
Points:	16	16	16	18	16	18	100
Score:							

$$\cosh^2(x) - \sinh^2(x) = 1, \quad \cosh^2(x) = \frac{1+\cosh(2x)}{2}, \quad \sinh(2x) = 2\sinh(x)\cosh(x)$$

$$\frac{d}{dx}(\sinh x) = \cosh x, \quad \frac{d}{dx}(\cosh x) = \sinh x, \quad \frac{d}{dx}(\tanh x) = \operatorname{sech}^2 x$$

$$\frac{d}{dx}(\coth x) = -\operatorname{csch}^2 x, \quad \frac{d}{dx}(\operatorname{sech} x) = -\operatorname{sech} x \tanh x, \quad \frac{d}{dx}(\operatorname{csch} x) = -\operatorname{csch} x \coth x$$

$$\frac{d}{dx}(\sinh^{-1} x) = \frac{1}{\sqrt{1+x^2}}, \quad \frac{d}{dx}(\cosh^{-1} x) = \frac{1}{\sqrt{x^2-1}}, \quad \frac{d}{dx}(\tanh^{-1} x) = \frac{1}{1-x^2}$$

$$\frac{d}{dx}(\operatorname{sech}^{-1} x) = \frac{-1}{x\sqrt{1-x^2}}, \quad \frac{d}{dx}(\coth^{-1} x) = \frac{1}{1-x^2}, \quad \frac{d}{dx}(\operatorname{csch}^{-1} x) = \frac{-1}{|x|\sqrt{1+x^2}}$$

1. (a) (8 points) Prove the following identity

$$\sinh^2(x) = \frac{\cosh(2x) - 1}{2}$$

(b) (8 points) Calculate the following integral

$$\int \sinh^2(x) \cosh^3(x)\, dx$$

2. Consider the differential equation

$$\frac{dy}{dx} = x^4 y^2.$$

(a) (12 points) Find the general solution to the differential equation

(b) (4 points) Find the particular solution satisfying $y(0) = 10$.

3. (a) (8 points) Evaluate the limit of the sequence $\lim_n \frac{n^2}{2e^n}$.

(b) (8 points) Use the squeeze theorem to calculate $\lim_n \frac{3n - \cos(n)}{2n}$.

4. Evaluate the series:

(a) (9 points) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n + 2^n}{7^{n-1}}$

(b) (9 points) $\displaystyle\sum_{n=3}^{\infty} \frac{1}{n(n-1)}$. Hint: Use partial fractions.

5. Determine whether the following series converge or diverge. Show all work to justify your answers.

(a) (8 points) $\displaystyle\sum_{n=1}^{\infty} \frac{2n-1}{n^4-n^2+7}$

(b) (8 points) $\displaystyle\sum_{n=1}^{\infty} e^{1/n^2}$

6. Determine whether the following series converge conditionally, converge absolutely, or diverge. Justify your answer.

(a) (9 points) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n+5}$

(b) (9 points) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \sin(n)}{n^2}$