Test 1
Math 222 Spring 2011
February 10, 2011
Name: Ivan Blank

Time of Recitation: 12:30

Initials of Recitation Instructor: IB

You may not use any type of calculator whatsoever. (Cell phones off and away!) You are not allowed to have any other notes, and the test is closed book. Use the backs of pages for scrapwork, and if you write anything on the back of a page which you want to be graded, then you should indicate that fact (on the front). Do not unstaple or remove pages from the exam.

By taking this exam you are agreeing to abide by KSU's Academic Integrity Policy.

Simple or standard simplifications should be made. You must **show your work** for every problem, and in order to get credit or partial credit, your work must make sense!

GOOD LUCK!!!

Problem	Possible	Score	Problem	Possible	Score
0	2	2	4	9	9
1	20	20	5	18	18
2	16	16	6	6	6
3	14	14	7	15	15
Total	52	52		48	48

1. Define $\vec{u} := (1, -2, 2)$ and $\vec{w} := (3, 4, -1)$. Compute the following:

(a) $|\vec{u}| = \sqrt{1+4+4} = \sqrt{9} = 3$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 2 \\ 3 & 4 & -1 \end{vmatrix}$$

(b) $\vec{u} \bullet \vec{w} = 3 - 8 - 2 = -7$

(c) $\vec{u} \times \vec{w} = (-6, 7, 10)$

(d) The area of the parallelogram spanned by $\vec{u}$ and $\vec{w}$.

$$= \sqrt{36 + 49 + 100} = \sqrt{185}$$

2. Find an equation for the plane which contains the points

$$\overbrace{(2,3,4)}^{A}, \overbrace{(1,-1,2)}^{B}, \overbrace{(-1,2,3)}^{C}.$$

$\vec{BA} = (1, 4, 2)$

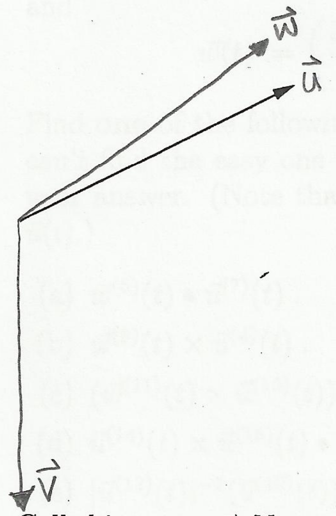
$\vec{CA} = (3, 1, 1)$

$$\vec{N} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 4 & 2 \\ 3 & 1 & 1 \end{vmatrix} = (2, 5, -11)$$

$$(x-2, y-3, z-4) \cdot (2, 5, -11) = 0$$

$2x + 5y - 11z = -25$

3. Here is a vector which you can assume has unit length:



Call this vector $\vec{u}$. Now using the same base point draw a vector $\vec{w}$ (and label it) so that the following are all satisfied:

(a) $|\vec{w}| = 1$.

(b) $\vec{u} \times \vec{w}$ points toward you.

(c) $0 < |\vec{u} \times \vec{w}| < 0.15$.

Next using the same base point again draw a vector $\vec{v}$ (and label it) so that the following are all satisfied:

(a) $|\vec{v}| = 1$.

(b) $-0.15 > \vec{u} \bullet \vec{v} > -0.85$.

(c) $\vec{u} \times \vec{v}$ points away from you.

4. Suppose

$$\vec{u}(t) := (3t^2e^{-4t^5}\sin(6t), 2t^3e^{-5t^4}\cos(7t), t\cos(\sin(e^{3t})))$$

and

$$\vec{w}(t) := \left(\frac{3+\ln(1+t^2)}{5+t^4}, t^5\sin(t^2), \frac{t}{1+t^6}\right).$$

Find **one** of the following. (Hint: Pick the easy one! Hint #2: If you can't find the easy one then skip this problem.) Explain how you got your answer. (Note that $\vec{u}^{(65)}(t)$ is notation for the 65$^{\text{th}}$ derivative of $\vec{u}(t)$.)

(a) $\vec{w}^{(5)}(t) \bullet \vec{u}^{(7)}(t)$.

(b) $\vec{w}^{(9)}(t) \times \vec{u}^{(4)}(t)$.

(c) $(\vec{w}^{(11)}(t) \times \vec{u}^{(15)}(t)) \bullet \vec{w}^{(11)}(t)$.

(d) $\vec{u}^{(14)}(t) \times \vec{u}^{(15)}(t) \bullet \vec{w}^{(14)}(t) \times \vec{w}^{(15)}(t)$.

(e) $|\vec{u}^{(12)}(t)|^{-1}(\vec{u}^{(12)}(t))$.

(C) Answer $= 0$

$\vec{w}^{(11)}(t) \times \vec{u}^{(15)}(t)$ is $\perp$ to $\vec{w}^{(11)}(t)$

and the dot product of

$\perp$ vec's $= 0$.

5. Match the equation to the surface. The equations are:

D (a) $y - z^2 = 0$.

E (b) $4x^2 + \frac{y^2}{9} + z^2 = 1$.

I (c) $z = x^2 + y^2$.

B (d) $x^2 + y^2 = z^2$.

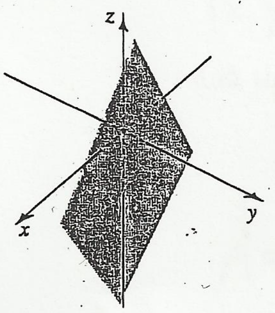
A (e) $2x + 3y - z = 5$.

G (f) $z^2 - 1 = x^2 + y^2$.

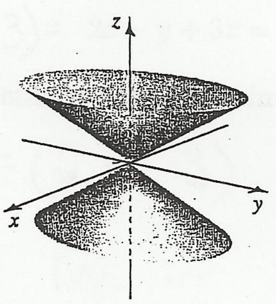
F (g) $x^2 + \frac{y^2}{9} - z^2 = 1$.

H (h) $z = x^2 - y^2$.

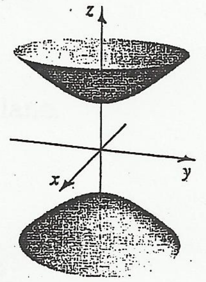
C (i) $y = |x|$.

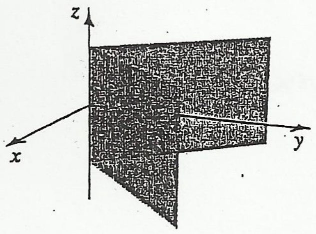


(A)

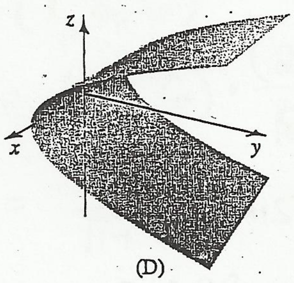


(B)

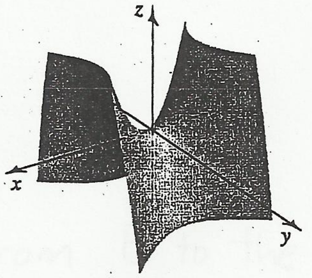
(G)


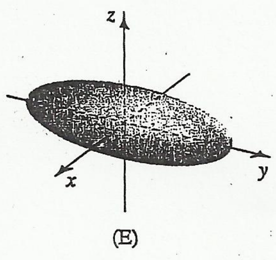


(C)

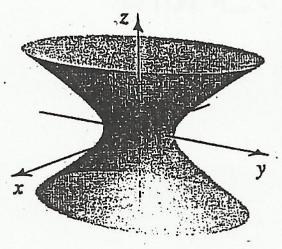


(D)

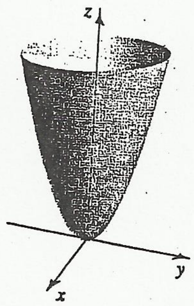
(H)




(E)



(F)

(I)


6. Suppose

$$\vec{u}(t) := (3t^4, \sin(5t)) \quad \text{and} \quad \vec{w}(t) := (\cos(7t), e^{6t}) .$$

Compute

$$\frac{d}{dt}(\vec{u}(t) \bullet \vec{w}(t)) = \vec{u}\,'(t) \bullet \vec{w}(t) + \vec{u}(t) \bullet \vec{w}\,'(t)$$

$$= (12t^3, 5\cos(5t)) \bullet (\cos(7t), e^{6t})$$

$$+ (3t^4, \sin(5t)) \bullet (-7\sin(7t), 6e^{6t})$$

$$= 12t^3\cos(7t) + 5e^{6t}\cos(5t) - 21t^4\sin(7t) + 6e^{6t}\sin(5t)$$

7. Write down a vector normal to the plane

$$(x,y,z)\cdot(2,-1,3) = 2x - y + 3z = 6 ,$$

and then find the distance from the point $\overbrace{(4,5,6)}^{B}$ to this plane.

$$\vec{N} = (2,-1,3)$$

$A = (3,0,0)$ lies in the plane

so $\vec{AB} = (1,5,6)$

$$\text{comp}_{\vec{N}} \vec{AB} = (1,5,6) \cdot \frac{(2,-1,3)}{|(2,-1,3)|}$$

$= \frac{15}{\sqrt{14}} =$ distance from B to the plane.