Test 3
Math 222 Spring 2011
April 14, 2011
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Time of Recitation: ______

Initials of Recitation Instructor: I. B.

You may not use any type of calculator whatsoever. (Cell phones off and away!) You are not allowed to have any other notes, and the test is closed book. Use the backs of pages for scrapwork, and if you write anything on the back of a page which you want to be graded, then you should indicate that fact (on the front). Do not unstaple or remove pages from the exam.

By taking this exam you are agreeing to abide by KSU's Academic Integrity Policy.

Simple or standard simplifications should be made. You must **show your work** for every problem, and in order to get credit or partial credit, your work must make sense!

GOOD LUCK!!!

Problem	Possible	Score	Problem	Possible	Score
0	2		4	15	
1	18		5	10	
2	15		6	15	
3	15		7	10	
Total	50			50	

1. Compute the mass of a solid E given by

$$0 \leq x \leq 2,\ 0 \leq y \leq 3,\ 0 \leq z \leq 5$$

and whose density function is given by $\rho(x, y, z) = 1 + xy^2z^4$.

$$\int_0^2 \int_0^3 \int_0^5 (1 + xy^2z^4)\, dz\, dy\, dx$$

$$= 30 + \frac{2^2}{2} \cdot \frac{3^3}{3} \cdot \frac{5^5}{5} = 30 + 2 \cdot 3^2 \cdot 5^4$$

2. Let R be the region given by the inequalities:

$$y \geq 0,\ z \geq 0,\ x^2 + y^2 + z^2 \leq 36\ .$$

Find

$$\iiint_R z^2\, dV\ .$$

$$= \int_{\theta=0}^{\pi} \int_{\phi=0}^{\frac{\pi}{2}} \int_{\rho=0}^{6} \rho^2 \cos^2\phi\ \rho^2 \sin\phi\ d\rho\, d\phi\, d\theta$$

$$= \pi \int_{\phi=0}^{\frac{\pi}{2}} \cos^2\phi\ \sin\phi\ d\phi \int_{\rho=0}^{6} \rho^4\, d\rho$$

$u = \cos\phi$

$du = -\sin\phi\ d\phi$

$$= \pi \cdot \frac{6^5}{5} \cdot \int_{u=1}^{0} -u^2\, du = \pi\ \frac{6^5}{5 \cdot 3}$$

3. Let R be the region given by the inequalities:

$$x \geq 0,\ x^2 + y^2 \leq 9,\ 0 \leq z \leq x^2 + y^2 .$$

Find

$$\iiint_R z\, dV .$$

$$= \int_{\theta=-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_{r=0}^{3} \int_{z=0}^{r^2} z \; dz\, r\, dr\, d\theta$$

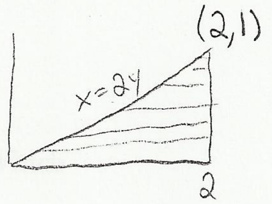
$$= \pi \int_{r=0}^{3} \frac{r^4}{2} \cdot r \; dr = \pi \int_{0}^{3} \frac{r^5}{2}\, dr$$

$$= \pi \left. \frac{r^6}{12} \right|_{0}^{3} = \pi \cdot \frac{3^5}{4}$$

4. Change the order of integration for the following two integrals:

(a)

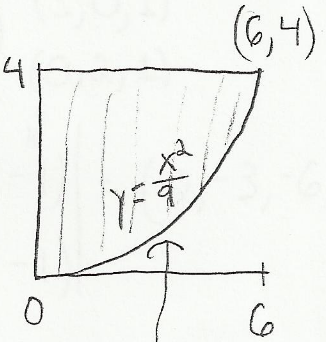
$$\int_{y=0}^{1}\int_{x=2y}^{2} \rho(x,y)\,dx\,dy\ .$$



$$= \int_{x=0}^{2}\int_{y=0}^{\frac{x}{2}} \rho(x,y)\,dy\,dx$$

(b)

$$\int_{x=0}^{6}\int_{y=\frac{x^2}{9}}^{4} \rho(x,y)\,dy\,dx\ .$$



$x^2 = 9y$

$x = 3\sqrt{y}$

$$\int_{y=0}^{4}\int_{x=0}^{3\sqrt{y}} \rho(x,y)\,dx\,dy$$

5. Let T be the tetrahedral with vertices

$$(3,2,1),\ (3,2,0),\ (3,0,1),\ (0,2,1)\ .$$

Set up the integral:

$$\iiint_T \rho(x,y,z)\, dV\ ,$$

as an iterated integral "dx dy dz." Multiple choice sub-question: Which of the following is the most relevant for your integration?

(a) The "shadow" or projection of T onto the xy-plane.

(b) The projection of T onto the xz-plane.

$\boxed{\text{(c)}}$ The projection of T onto the yz-plane.

$x \le 3$
$y \le 2$
$z \le 1$

Plane containing $\left\{ \begin{array}{l} (3,2,0) \\ (3,0,1) \\ (0,2,1) \end{array} \right.$

has vec's $(0,2,-1)$ & $(3,0,-1)$: $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 2 & -1 \\ 3 & 0 & -1 \end{vmatrix} = (-2,-3,-6)$

$\vec{N} = (2,3,6)$ works Plane $= \{ (2,3,6)\cdot(x,y-2,z-1) = C$

Which is $2x+3y+6z = 12$

Using $(3,2,1) \Rightarrow 2x+3y+6z \ge 12$

$$\frac{12-3y-6z}{2} \le x \le 3$$

$$2-2z = \frac{12-6-6z}{3} \le \frac{12-2x-6z}{3} \le y \le 2$$

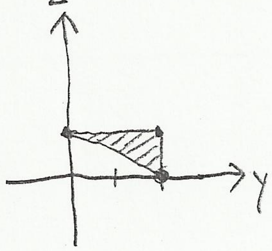
max over all x

$$\int_{z=0}^{1} \int_{y=2-2z}^{2} \int_{x=\frac{12-3y-6z}{2}}^{3} \rho(x,y,z)\, dx\, dy\, dz$$

yz proj

(0,2,1)
(0,2,0)
(0,0,1)
(0,2,1)

Same!



$\hat{\imath}$ $\hat{\jmath}$ $\hat{k}$

∂_x ∂_y ∂_z

6. Find the curl and the divergence of the following vector fields:

(a) $\vec{F}(x,y,z) := (xyz,\ 2xy+3xz+4yz,\ 5x+6y+7z)$.

$\nabla\cdot\vec{F} = yz + 2x + 4z + 7$

$\nabla\times\vec{F} = (6-3x-4y,\ xy-5,\ 2y+3z-xz)$

$\hat{\imath}$ $\hat{\jmath}$ $\hat{k}$

∂_x ∂_y ∂_z

(b) $\vec{G}(x,y,z) := (\sin(xy),\ \cos(yz),\ e^{xz})$.

$\nabla\cdot\vec{G} = y\cos(xy) - z\sin(yz) + xe^{xz}$

$\nabla\times\vec{G} = (y\sin(yz),\ -ze^{xz},\ -x\cos(xy))$

7. Which one of the following vector fields is conservative? Find it, and find a corresponding potential function.

(a) $\vec{F}(x,y,z) = (2xy - y^4,\ x^2 - 3xy^3,\ 0)$.

$\boxed{\text{(b)}}$ $\vec{G}(x,y,z) = (2xy - y^3,\ x^2 - 3xy^2,\ 0)$.

(c) $\vec{H}(x,y,z) = (2xy - y^2,\ x^2 - 3xy,\ 0)$.

y-deriv. comp. 1	x-deriv. Comp 2
$2x - 4y^3$	$2x - 3y^3$
$2x - 3y^2$	$2x - 3y^2$
$2x - 2y$	$2x - 3y$

$$\vec{G} = \nabla g$$

$$g(x,y,z) = \int (2xy - y^3)\,dx = x^2y - xy^3 + P(y,z)$$

$$= \int (x^2 - 3xy^2)\,dy = x^2y - xy^3 + Q(x,z)$$

$$= \int 0\,dz = R(x,y)$$

$$g(x,y,z) = x^2y - xy^3 + \text{Const.}$$

or

$$\frac{\partial}{\partial y}\left[g(x,y,z) = \int (2xy - y^3)\,dx = x^2y - xy^3 + P(y,z)\right]$$

$$\hookrightarrow g_y(x,y,z) = x^2 - 3xy^2 + P_y(y,z) = x^2 - 3xy^2$$

$$\Rightarrow P_y(y,z) = 0 \Rightarrow P(y,z) = Q(z)$$

$$g_z(x,y,z) = Q'(z) = 0 \Rightarrow Q(z) = \text{Const.}$$

$$\Rightarrow g(x,y,z) = x^2y - xy^3 + \text{Const.}$$