Math 222: Analytic geometry and Calculus 3
Exam 1: Summer 2011
June 21, 2011

Name...........................
Instructor....................

To receive credit you **must** show your work

(20 pts) Problem 1. The points P(1,1,1), Q(2,0,3) and R(2,-2,-1) determine a triangle in 3-space.

a) Find the angle at the vertex P in degree

b) find the area of the triangle

c) find the equation of the plane which contains the triangle

d) find symmetric equations of the line which passes through the point R and is orthogonal (perpendicular) to the plane containing the triangle

(10 pts) Problem 2.

a) Find parametric equations for the line which passes through the point $(1, -1, 1)$ and is orthogonal (perpendicular) to the plane $x + 2y + 3z = 16$

b) Find the point where the line you found in part a) intersects the plane:

$$x + 2y + 3z = 16.$$

c) Using the result in part b) find the distance from the point $(1, -1, 1)$ to the plane $x + 2y + 3z = 16$.

(30 pts) Problem 3. An object moving in 3-space according to the parametric equations

$$x = 2t + 1, \quad y = \cos(t), \quad z = t^2 + 2$$

where t is the time.

Find, as function of t,

a) position vector $\mathbf{r} =$

b) velocity vector $\mathbf{v} =$

c) acceleration vector $\mathbf{a} =$

d) speed $v =$

e) $a_T =$

f) find the curvature at $t = 0$

g) find the normal component a_N at $t = 0$

(15 pts) Problem 4.

An object is moving 3-space in such a way that its acceleration vector as a function of time t is

$$\mathbf{a} = \langle -\cos(t) + t, e^{-t} + 2, \sin(t) \rangle.$$

At time $t = 0$ its velocity vector and its position vector are given by

$$\mathbf{v}(0) = \langle 0, -1, 0 \rangle, \qquad \mathbf{r}(0) = \langle 2, 1, 0 \rangle,$$

respectively.

a) Find the velocity vector as a function of t

b) find the position vector as a function of t

c) give the parametric equations for the motion

(10 pts) Problem 5.

Let

$$\mathbf{u} = \langle 1, 2, -1 \rangle, \qquad \mathbf{v} = \langle 1, 0, 1 \rangle.$$

a) Find two unit vectors which are orthogonal to both $\mathbf{u}$ and $\mathbf{v}$.

b) Find the vector projection of $\mathbf{v}$ onto $\mathbf{u}$, $proj_{\mathbf{u}}\mathbf{v} =$

(15 pts) Problem 6.

a) Find the length of the curve:

$$\mathbf{r}(t) = \left\langle 2t, t^2, \frac{1}{3}t^3 \right\rangle, \qquad 0 \le t \le 1.$$

b) Calculate the following expression

$$\left\langle \left(\frac{e^{\sin(x)}}{1+\tan^4(x)}\right)', (e^{x^2})', (\cos(e^x))' \right\rangle \bullet \left(\left\langle \left(\frac{e^{\sin(x)}}{1+\tan^4(x)}\right)', (e^{x^2})', (\cos(e^x))' \right\rangle \times \langle 1, -1, 3 \rangle \right)$$

c) Let u, v, w be vectors in 3-space. Which of the following expressions makes sense, which doesn't

(1) $u \times (v \bullet w)$
(2) $u \bullet (v \times w)$
(3) $u \times (v \times w) + 2$
(4) $u \bullet (v \bullet w)$
(5) $u + (v \bullet w)$