Math 222: Analytic geometry and Calculus 3
Exam 1: Summer 2011
July 13, 2011

Name..........................
Instructor...................

To receive credit you **must** show your work

(20 pts) Problem 1. Let $P(2, 1, -2)$, $Q(1, 0, -1)$ and

$$f(x, y, z) = \sqrt{x^2 + y^2 + z^2}.$$

a) Find the directional derivative of f at P in the direction of Q

b) find the maximum rate of change of f at the point P and the direction in which it occurs

c) find the tangent plane to the surface $f(x, y, z) = 3$ at the point P

d) find the linear approximation to the function f at the point P and use it to approximate $f(1.99, 1, -1.98)$.

(15 pts) Problem 2.
Find the **local** maximum and minimum values and saddle points of the function

$$f(x, y) = x^3 + 3xy^2 + 3x^2 + 3y^2.$$

(10 pts) Problem 3.

Use the method of **Lagrange multipliers** to find the maximum and minimum values of the function

$$f(x) = x + 2y - 3z + 2,$$

subject to the given constraint

$$x^2 + y^2 + z^2 = 14.$$

(15 pts) Problem 4.

Use a **double integral** to find the volume of the solid which is bounded by the surface $z = 1 + 8xy$ and the planes $z = 1$, $y = x$, $x = 0$, $y = 1$.

(20 pts) Problem 5.

Use **cylindrical coordinates** to set up integrals (but DO NOT solve, i.e., DO NOT compute the integrals) for computing:

a) the **volume** of the solid which is bounded by the surfaces $z = 2 - x^2 - y^2$, $z = x^2 + y^2$.

b) the **mass** of the solid which lies **in the first octant** and is bounded by the surfaces $z = 2 - x^2 - y^2$, $z = \sqrt{x^2 + y^2}$ with density function $\delta(x, y, z) = \sqrt{x^2 + y^2}$.

(20 pts) Problem 6.

Use spherical coordinates to set up integrals (but DO NOT solve, i.e., DO NOT compute the integrals) for computing:

a) the **mass** of the solid which is bounded by the surfaces $z = \sqrt{x^2 + y^2}$, $x^2 + y^2 + z^2 = 1$, and $z = 2$ with density function $\delta(x, y, x) = \sqrt{x^2 + y^2 + z^2}$.

b) the **volume** of the solid which is **in the first octant** and is bounded by the surfaces $z = \sqrt{x^2 + y^2}$, $x^2 + y^2 + z^2 = 9$, and $z = 1$.