Math 222 **Exam 1** **September 25, 2014**

Instructions: Wait to open the exam until instructed to do so. Then answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page. You will have 1 hour to complete this exam.

Question	Points	Score
1	30	
2	10	
3	10	
4	10	
5	10	
6	20	
7	10	
Total:	100	

Name: ______________________________

Recitation Instructor: ______________________________

1. For the following questions, suppose $\mathbf{u} = \langle 1, -1, 1 \rangle$ and $\mathbf{v} = \langle 0, 2, 1 \rangle$.

   (a) (4 points) Evaluate $\mathbf{u} - \mathbf{v}$.

   (b) (4 points) Evaluate $\mathbf{u} \cdot \mathbf{v}$.

   (c) (4 points) Evaluate $\mathbf{u} \times \mathbf{v}$.

   (d) (4 points) Find the volume of the parallelepiped spanned by $\mathbf{i}$, $\mathbf{u}$ and $\mathbf{v}$.

(e) (4 points) Find a vector orthogonal to $\mathbf{u} + \mathbf{v}$ and $\mathbf{u} - \mathbf{v}$.

(f) (5 points) Find a parametric equation for the line ℓ passing through $(-1, 0, 1)$ and parallel to $\mathbf{v}$.

(g) (5 points) Find the equation for the plane containing $(1, 0, -1)$ and parallel to both $\mathbf{u}$ and $\mathbf{v}$.

2. (a) (5 points) Find an equation for $xz = y$ of the form $\rho = f(\theta, \phi)$ in spherical coordinates.

   (b) (5 points) Recall that spherical coordinates (ρ, θ, ϕ) must satisfy $0 \leq \rho$, $0 \leq \theta < 2\pi$ and $0 \leq \phi \leq \pi$. Describe the domain of the function f found in part (a).

3. (10 points) Parametrize each component of the intersection of the surfaces $xyz = 1$ and $z = xy$.

4. (10 points) Find all points on the path

$$\mathbf{r}(t) = \left\langle e^t + t^2, 2\ln(1+t^2) - t^2, t^3 \right\rangle$$

whose tangent line is parallel to the xz-plane.

5. (10 points) Compute the length of the path

$$\mathbf{r}(t) = \left\langle e^t, 2e^t, e^{2t} \right\rangle$$

over the interval $0 \le t \le \ln(\sqrt{5})$.

6. Answer the following questions concerning the path

$$\mathbf{r}(t) = \int_0^t \langle 5\cos(e^s), 3\sin(e^s), 4\sin(e^s)\rangle \, \mathrm{d}s.$$

(a) (5 points) Find the unit tangent vector $\mathbf{T}(t)$.

(b) (5 points) Find the unit normal vector $\mathbf{N}(t)$.

(c) (5 points) Find the curvature $\kappa(t)$.

(d) (5 points) Find the coefficients $a_{\mathbf{T}}$ and $a_{\mathbf{N}}$ in the linear combination $a_{\mathbf{T}}\mathbf{T} + a_{\mathbf{N}}\mathbf{N}$.

7. (10 points) Draw a contour map of $f(x, y) = e^x \cos(y)$ using at least four level curves.

Coordinate systems

Cylindrical

$$x = r\cos(\theta)$$
$$y = r\sin(\theta)$$
$$z = z$$

$$r = \sqrt{x^2 + y^2}$$
$$\theta = \tan^{-1}(y/x)$$
$$z = z$$

Spherical

$$x = \rho\cos(\theta)\sin(\phi)$$
$$y = \rho\sin(\theta)\sin(\phi)$$
$$z = \rho\cos(\phi)$$

$$\rho = \sqrt{x^2 + y^2 + z^2}$$
$$\theta = \tan^{-1}(y/x)$$
$$\phi = \cot^{-1}\left(\frac{z}{\sqrt{x^2 + y^2}}\right)$$

Unit vectors and curvature

$$\mathbf{T} = \frac{\mathbf{r}'(t)}{\|\mathbf{r}'(t)\|}$$

$$\mathbf{N} = \frac{\mathbf{T}'(t)}{\|\mathbf{T}'(t)\|}$$

$$\kappa(t) = \frac{\|\mathbf{r}'(t) \times \mathbf{r}''(t)\|}{\|\mathbf{r}'(t)\|^3}$$

$$v(t) = \|\mathbf{r}'(t)\|$$

$$\mathbf{T}'(t) = v(t)\kappa(t)\mathbf{N}(t)$$

Acceleration components

$$\mathbf{a}(t) = a_{\mathbf{T}}(t)\mathbf{T}(t) + a_{\mathbf{N}}(t)\mathbf{N}(t)$$

$$a_{\mathbf{T}} = v'(t) = \mathbf{a} \cdot \mathbf{T}$$

$$a_{\mathbf{T}}\mathbf{T} = \frac{\mathbf{a} \cdot \mathbf{v}}{\mathbf{v} \cdot \mathbf{v}}\mathbf{v}$$

$$a_{\mathbf{N}} = \kappa(t)v(t)^2$$

$$a_{\mathbf{N}}\mathbf{N} = \mathbf{a} - a_{\mathbf{T}}\mathbf{T}$$