Math 222 **Exam 2** **October 23, 2014**

Instructions: Wait to open the exam until instructed to do so. Then answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page. You will have 1 hour and 20 minutes to complete this exam.

Question	Points	Score
1	15	
2	15	
3	10	
4	15	
5	15	
6	10	
7	10	
8	10	
Total:	100	

Name: ______________________________

Recitation Instructor: ______________________________

Recitation Time: ______________________________

1. Find the limit, if it exists. If the limit does not exist, explain why.

(a) (5 points)

$$\lim_{(x,y)\to(1,-1)} \left(\sin(\pi e^{x+y}) + \frac{x^2+y^2}{x-y} \right)$$

(b) (5 points)

$$\lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2+y^2}}$$

(c) (5 points)

$$\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x^2+y^2}$$

2. Evaluate the partial derivatives, if they exist. If they do not exist, explain why.

   (a) (5 points) $f_x(1, 2, 3)$ for $f(x, y, z) = z\sin(\pi yx) + e^{zx-1}$.

   (b) (5 points) $f_{xyz}(1, 1, 1)$ for $f(x, y, z) = e^{\ln(x^2+y^2)-x^{3/100}} + \cos(z^{\sin(x)}) - \tan^{-1}(y^2 - z^2)$

   (c) (5 points) $f_{xy}(0, 0)$ for $f(x, y) = (x^2 + y^2)^{3/2}$.

3. Consider the function $f(x, y) = x^2 + y^2 + \ln(x + y)$.

   (a) (5 points) Give the linear approximation for $f(x, y)$ at $(1/2, 1/2)$.

   (b) (5 points) Use your approximation to give an estimate for the change in f

$$\Delta f = f(1/3, 3/5) - f(1/2, 1/2).$$

4. Let

$$f(x, y, z) = xye^z$$

(a) (5 points) Find the gradient of f.

(b) (5 points) Give the equation for the tangent plane of the level surface

$$f(x, y, z) = -1$$

at $P = (1, -1, 0)$.

(c) (5 points) Give an example of a vector $\mathbf{v}$ for which f is decreasing in the direction of $\mathbf{v}$ starting at $(1, -1, 0)$.

5. Let

$$f(x,y) = e^{x^3-3x+y^2}$$

(a) (5 points) Find the critical points of f.

(b) (5 points) Describe the local behavior of f near the critical points.

(c) (5 points) Find a global maximum value or a global minimum value for f if it exists. Explain your response.

6. (10 points) Use Lagrange multipliers to find the critical points of the function $f(x, y, z) = xyz$ on the unit sphere $x^2 + y^2 + z^2 = 1$. Identify the global maximum value and the global minimum value of f on the sphere.

7. (10 points) Let D be the unit disc centered at the origin. Evaluate the double integral

$$\iint_D (y^3 - x^3)\ \mathrm{d}A$$

8. (10 points) Let

$$D = \{(x,y) : x \geq 0, y \geq -1, x + y \leq 1\}.$$

Evaluate the double integral

$$\iint_D (y - 2y^2)e^{xy}\ \mathrm{d}A$$