Instructions: Wait to open the exam until instructed to do so. Then answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page. You will have 1 hour and 20 minutes to complete this exam.

Question	Points	Score
1	10	
2	15	
3	15	
4	15	
5	15	
6	15	
7	15	
Total:	100	

Name: ______________________________

Recitation Instructor: ______________________________

Recitation Time: ______________________________

1. (10 points) Calculate the integral

$$\iiint_{\mathcal{B}} x^2 y \cos(xyz)\ \mathrm{d}V$$

where $\mathcal{B} = [0, \pi] \times [0, 1] \times [0, 1]$.

2. (15 points) Calculate the integral of

$$f(x, y, z) = 12xyz$$

over the region

$$\mathcal{W}:\ 0 \leq x,\ 0 \leq y,\ 0 \leq z \leq 1 - x^2 - y^2$$

3. Consider the region

$$\mathcal{R}: \ y \geq x, \ x^2 + y^2 \leq 4$$

(a) (10 points) Express

$$\iint_{\mathcal{R}} e^{x^2+y^2} \ \mathrm{d}A$$

as an iterated integral using polar coordinates.

(b) (5 points) Evaluate the integral.

4. Let $\mathcal{P}$ be the parallelogram in the plane spanned by the vectors $\langle 1, -1 \rangle, \langle 1, 1 \rangle$.

   (a) (5 points) Find a linear mapping $G(u, v)$ which maps the square $[0, 1] \times [0, 1]$ to $\mathcal{P}$.

   (b) (5 points) Compute the Jacobian of G.

(c) (5 points) If

$$\int_{\mathcal{P}} f(x,y)\ \mathrm{d}A = \pi$$

then compute

$$\int_0^1 \int_0^1 f(x(u,v), y(u,v))\ \mathrm{d}u\ \mathrm{d}v$$

5. (15 points) Calculate

$$\int_{\mathcal{C}} x e^{\cos(y) - z} \, \mathrm{d}s$$

where $\mathcal{C}$ is the helix parametrized by $\mathbf{c}(t) = (\sin(t), t, \cos(t))$ for $0 \leq t \leq \pi$.

6. (15 points) Evaluate

$$\int_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{s}$$

where $\mathbf{F}(x, y, z) = \langle e^x, e^y, e^{x+y} \rangle$ and $\mathcal{C}$ is the curve

$$\mathbf{c}(t) = (\ln(t-1), \ln(t+1), \ln(t^2-1))$$

for $2 \le t \le 4$.

7. Consider the vector field

$$\mathbf{F}(x, y, z) = \langle y + z, x + z, x + y \rangle$$

and $\mathbf{c}(t) = (\sqrt{t}, \sin(\pi t^2), e^t)$ for $0 \leq t \leq 4$.

(a) (5 points) Show that $\mathbf{F}$ satisfies the cross-partials condition.

(b) (5 points) Find a potential function for $\mathbf{F}$ or determine that $\mathbf{F}$ is not conservative. Explain your response.

(c) (5 points) Evaluate $\int_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{s}$.

Coordinate systems

Polar	Cylindrical	Spherical
$x = r\cos(\theta)$	$x = r\cos(\theta)$	$x = \rho\cos(\theta)\sin(\phi)$
$y = r\sin(\theta)$	$y = r\sin(\theta)$	$y = \rho\sin(\theta)\sin(\phi)$
	$z = z$	$z = \rho\cos(\phi)$
$r = \sqrt{x^2+y^2}$	$r = \sqrt{x^2+y^2}$	$\rho = \sqrt{x^2+y^2+z^2}$
$\theta = \tan^{-1}(y/x)$	$\theta = \tan^{-1}(y/x)$	$\theta = \tan^{-1}(y/x)$
	$z = z$	$\phi = \cot^{-1}\left(\dfrac{z}{\sqrt{x^2+y^2}}\right)$
$\mathrm{d}x\,\mathrm{d}y = r\,\mathrm{d}r\,\mathrm{d}\theta$	$\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z = r\,\mathrm{d}r\,\mathrm{d}\theta\,\mathrm{d}z$	$\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z = \rho^2\sin\phi\,\mathrm{d}\rho\,\mathrm{d}\phi\,\mathrm{d}\theta$

Change of variables

$$G : \mathcal{D}_0 \to \mathcal{D}$$

$$G(u,v) = (x(u,v), y(u,v))$$

$$\mathrm{Jac}(G) = \frac{\partial x}{\partial u}\frac{\partial y}{\partial v} - \frac{\partial x}{\partial v}\frac{\partial y}{\partial u}$$

$$\int_{\mathcal{D}} f(x,y)\,\mathrm{d}x\,\mathrm{d}y = \int_{\mathcal{D}_0} f(x(u,v),y(u,v))\,\left|\mathrm{Jac}(G)\right|\,\mathrm{d}u\,\mathrm{d}v$$

Line integrals

$\mathbf{c}(t)$ for $a \le t \le b$ parametrizing $\mathcal{C}$

$$\int_{\mathcal{C}} f(x,y,z)\,\mathrm{d}s = \int_a^b f(\mathbf{c}(t))\,\|\mathbf{c}'(t)\|\,\mathrm{d}t$$

$$\int_{\mathcal{C}} \mathbf{F}\cdot\mathrm{d}\mathbf{s} = \int_a^b \mathbf{F}(\mathbf{c}(t))\cdot\mathbf{c}'(t)\,\mathrm{d}t$$