MATH 222 CALCULUS 3 SUMMER 2014: FINAL

Name: _____

Instructor: _____

To receive credit you must show your work.

Problem 1. (30 points) Compute the limit, or show it doesn't exist:

$$\lim_{(x,y)\to(0,0)} \frac{x^2y^2}{x^4+y^4}.$$

Problem 2. (30 points) Compute the limit, or show it doesn't exist:

$$\lim_{(x,y)\to(0,0)} sin(\frac{x^3 + 2\pi y^4 sin(x)}{x^3 + y^3})(y^{15} + 2x^{23}).$$

Problem 3. (30 points) $\mathcal{R}$ is the region bounded by the circle $x^2 + y^2 = 1$. Classify the critical points inside $\mathcal{R}$ (without the boundary) and find the largest and the smallest value of $f(x, y) = x^4 + y^4$ in $\mathcal{R}$ (including the boundary).

Problem 4. Do the required transformations, but **DO NOT CALCULATE**.

a)(10 points) Change the order of the integration:

$$\int_0^1 \int_{-x^2}^{x^2} \frac{sin(x+y^2)}{cos^3(xy)} dydx.$$

Do the required transformations, but **DO NOT CALCULATE**.

b)(10 points) Use spherical coordinates to state the integration:

$$\int\int\int_V \sqrt{x^2+y^2+z^2}dxdydz$$

where V is the region bounded by $x^2+y^2+z^2 \leq 4$ and $z \geq \sqrt{x^2+y^2}$.

Do the required transformations, but **DO NOT CALCULATE**.

c)(10 points) Let $u = x + y$ and $v = x - y$. Use change of variables formula to write down the integration:

$$\int\int_D (x^2 - y^2) dxdy$$

where D is the region bounded by $-1 \leq x + y \leq 4$ and $-2 \leq x - y \leq 3$.

Problem 5. (30 points) Compute the area of the region bounded by

$$\frac{(x-1)^2}{4} + \frac{(y-1)^2}{25} = 1.$$

Problem 6. (30 points) Compute the area of the parallelogram bounded by $y = 2x$, $y = \frac{1}{2}x$, $y = 2x - 3$ and $y = \frac{1}{2}x + \frac{3}{2}$.

Problem 7. (30 points) Find out the equation of the tangent plane of surface $x^2 + y^2 - z^2 - 6x + 4z + 4 = 0$ at the point $(4, 2, 4)$.

Problem 8. (30 points) Compute the line integral:

$$\oint_C (\sqrt{1+x^5}+2y)dx + (4x - e^{y^2})dy,$$

where C is $x^2 + y^2 = 4$ oriented counterclockwise.

Problem 9. (30 points) Compute the line integral:

$$\int_C (x^3 + y + 2z)dx + (x + 3z)dy + (2x + 3y)dz,$$

where C is $x = 2sin(t)$, $y = 2cos(t)$, $z = 3t$ and t is from 2π to 0.

Problem 10. (30 points) Compute the line integral:

$$\oint_C x dx + z dy,$$

where C is the triangle whose vertices are $(1,0,0)$, $(0,1,0)$ and $(0,0,1)$, and the orientation is $(1,0,0) \to (0,1,0) \to (0,0,1) \to (1,0,0)$.