Midterm Exam II
Math 222 Summer 2015
July 2, 2015

Name:

Instructor's Name:

Problem(1) [10 points]: Show that the limit of the function $f(x,y) = \frac{xy}{x^2+y^2}$ does not exist at $(0,0)$.

Evaluating limit along x-axis ($y=0$)

$$\lim_{(x,y)\to(0,0)} f(x,y) = \lim_{(x,0)\to(0,0)} f(x,0) = \lim_{x\to 0} \frac{0}{x^2+0}$$
$$= \lim_{x\to 0} 0 = \boxed{0}$$

+5

Evaluating limit along line $y=x$

$$\lim_{(x,y)\to(0,0)} f(x,y) = \lim_{(x,x)\to(0,0)} f(x,x) = \lim_{x\to 0} \frac{x\cdot x}{x^2+x^2}$$
$$= \lim_{x\to 0} \frac{\cancel{x^2}}{2\cancel{x^2}}$$
$$= \lim_{x\to 0} \frac{1}{2}$$
$$= \boxed{\frac{1}{2}}$$

+5

$\therefore$ limit doesn't exist at $(0,0)$.

Problem(2) [15 points]: Find the first partial derivatives of the following functions(You **do not** need to simplify):

a) $f(x, y) = 5x^3 sin(y) + 3x^2y^3 - 4ylnx$

$$f_x(x,y) = 15x^2\sin(y) + 6xy^3 - \frac{4y}{x}$$ +2

$$f_y(x,y) = 5x^3\cos(y) + 9x^2y^2 - 4\ln x$$ +2

b) $f(x, y, z) = xy^2z^3 + 3yz + \ln(x + 2y + 3z)$

$$f_x = y^2z^3 + \frac{1}{\ln(x+2y+3z)}$$ +2

$$f_y = 2xyz^3 + 3z + \frac{2}{\ln(x+2y+3z)}$$ +2

$$f_z = 3xy^2z^2 + 3y + \frac{3}{\ln(x+2y+3z)}$$ +2

c) $z = \sqrt{3x - 1} + \sin^2(x + 2y) + e^{x^2y}$

$$\frac{\partial z}{\partial x} = \frac{1}{2}(3x-1)^{-\frac{1}{2}} \cdot 3 + 2\sin(x+2y)\cdot\cos(x+2y)\cdot 1 + e^{x^2y}\cdot 2xy$$ +

$$\frac{\partial z}{\partial y} = 2\sin(x+2y)\cdot\cos(x+2y)\cdot 2 + e^{x^2y}\cdot x^2.$$ +2

Problem(3) [12 points]: Let $f(x,y) = x^2y^3 - 4y$ do the following:

a)Find the directional derivative of $f(x,y)$ at the point $P(2,-1)$ in the direction of the vector $\vec{v} = 2\vec{i}$ - $1\vec{j}$

$\nabla f = \langle 2xy^3, \ 3x^2y^2 - 4 \rangle$

$\nabla f_P = \langle 2\cdot 2(-1)^3, \ 3(2)^2(-1)^2 - 4 \rangle = \langle -4, 8 \rangle$ +2

$\vec{V} = \langle 2, -1 \rangle$ so, $\vec{u} = \frac{\vec{V}}{\|\vec{V}\|} = \frac{\langle 2, -1 \rangle}{\sqrt{4+1}} = \frac{\langle 2, -1 \rangle}{\sqrt{5}}$ +2

$\therefore D_{\vec{u}} f_P = \nabla f_P \cdot \vec{V}$

$= \langle -4, 8 \rangle \cdot \frac{\langle 2, -1 \rangle}{\sqrt{5}}$ +2

$= \frac{-8-8}{\sqrt{5}} = \boxed{\frac{-16}{\sqrt{5}}}$ +2

b) In what direction is the function increasing the fastest at $P(2,-1)$ and what is the rate of change in that direction?

+2 The function increasing the fastest in $\langle -4, 8 \rangle$ dir^n at P

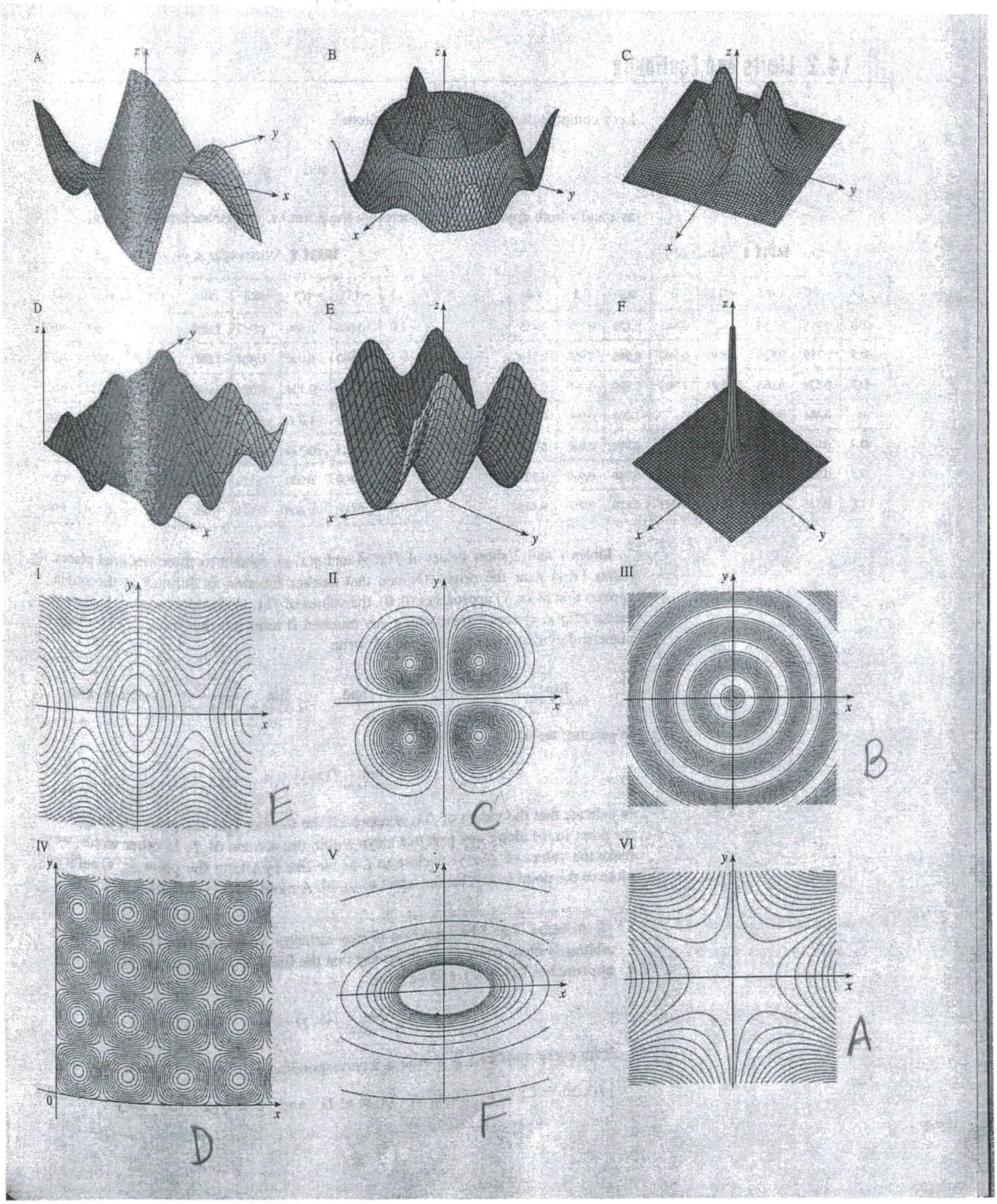
and rate of change in that direction $= \|\langle -4, 8 \rangle\|$

$= \sqrt{16+64}$

$= \boxed{\sqrt{80}}$ +2

or, $\boxed{4\sqrt{5}}$

Problem(4) [12 points]: Match the following graphs (labeled A - F) with the contour maps (labeled I-IV)



Problem(5) [12 points]: Use the linear approximation to $f(x, y) = \sqrt{\frac{x}{y}}$ at (9, 4) to estimate $\sqrt{\frac{9.1}{3.9}}$

$f(x,y)\ \sqrt{\frac{x}{y}} = x^{\frac{1}{2}} y^{-\frac{1}{2}} \Rightarrow f(9,4) = \frac{3}{2}$ −3

$f_x(x,y) = \frac{1}{2} x^{-\frac{1}{2}} y^{-\frac{1}{2}} \Rightarrow f_x(9,4) = \frac{1}{12}$ +3

$f_y(x,y) = -\frac{1}{2} x^{\frac{1}{2}} y^{-\frac{3}{2}} \Rightarrow f_y(9,4) = -\frac{3}{16}$ +3

We have, $h = 0.1, \quad k = -0.1$

Thus, $f(a+h, b+k) = f(a,b) + f_x(a,b)h + f_y(a,b)k$

gives,

$$\sqrt{\frac{9.1}{3.9}} \approx \frac{3}{2} + \frac{1}{12}(0.1) - \frac{3}{16}(-0.1)$$

$$= \frac{3}{2} + \frac{1}{120} + \frac{3}{160}$$

$$= \frac{720 + 4 + 9}{480}$$

$$= \boxed{\frac{733}{480}} \text{ or } \boxed{1.53}$$

+3

Problem(6) [10 points] Use chain rule to find the partial derivative $\frac{\partial f}{\partial r}$: $f(x, y, z) = xy + z^2, x = r + s - 2s, y = 3rt, z = s^2$

$$\frac{\partial f}{\partial r} = \frac{\partial f}{\partial x} \cdot \frac{\partial x}{\partial r} + \frac{\partial f}{\partial y} \frac{\partial y}{\partial r} + \frac{\partial f}{\partial z} \cdot \frac{\partial z}{\partial r}$$

$$= y \cdot 1 + x \cdot 3t + 2z \cdot 0$$

$$= 3rt + (r-s)3t$$

$$= 3rt + 3rt - 3st$$

$$= \boxed{6rt - 3st}$$

Problem(7) **[18 points]**: Find the maximum and minimum of $f(x, y) = x^2 - 4x + y^2 + 2y$ in the region where $x^2 + y^2 \leq 20$.

part I, $f(x,y) = x^2 - 4x + y^2 + 2y$ over $x^2 + y^2 < 20$

+1 $f_x = 2x - 4 = 0 \Rightarrow x = 2$

+1 $f_y = 2y + 2 = 0 \Rightarrow y = -1$

and $(2,-1)$ is in the domain

so $(2,-1)$ is critical point +2

$f(2,-1) = 4 - 8 + 1 - 2 = -5$ $\boxed{\text{Minimum}}$ +1

Part II, $f(x,y) = x^2 - 4x + y^2 + 2y$ over $x^2 + y^2 = 20$

$g(x,y) := x^2 + y^2 - 20$

$\nabla f = \lambda \nabla g$

$\langle 2x - 4, 2y + 2 \rangle = \lambda \langle 2x, 2y \rangle$ +2

$2x - 4 = \lambda 2x \Rightarrow \lambda = \frac{2x-4}{2x} = \frac{x-2}{x}$ — (I)

$2y + 2 = \lambda 2y \Rightarrow \lambda = \frac{2y+2}{2y} = \frac{y+1}{y}$ — (II)

$x^2 + y^2 = 20$ — (III)

From (I) and (II) $\frac{x-2}{x} = \frac{y+1}{y} \Rightarrow \cancel{xy} - 2y = \cancel{xy} + x$

$\Rightarrow x = -2y$

From (III) $(-2y)^2 + y^2 = 20$

$y^2 = 4$ when $y = 2$, $x = -4$

$y = \pm 2$ when $y = -2$, $x = 4$

$\therefore$ $(-4,2)$, $(4,-2)$ are critical points +6

$f(-4,2) = 16 + 16 + 4 + 4 = 40$ — $\boxed{\text{Maximum}}$ +2

$f(4,-2) = 16 - 16 + 4 - 4 = 0$ — +2

Problem(8) [11 points]: Set up, **but do not solve** the following Lagrange Multipliers problems.

Minimize: $F(x, y, z) = xe^{2y+3z}$ Subject to the constraint: $G(x, y, z) = x^4 + y^4 + z^4xy + 2xz - 3yz = 40$.

$\nabla F = \lambda \nabla G$ +1

+4 $\langle e^{2y+3z},\ 2xe^{2y+3z},\ 3xe^{2y+3z} \rangle = \lambda \langle 4x^3 + z^4y + 2z,$

$4y^3 + z^4x - 3z,\ 3xyz^3 + 2x - 3$

+1 $e^{2y+3z} = \lambda(4x^3 + z^4y + 2z)$

+1 $2xe^{2y+3z} = \lambda(4y^3 + z^4x - 3z)$

+1 $3xe^{2y+3z} = \lambda(4xyz^3 + 2x - 3y)$

+1 $x^2 + y^2 + z^2xy + 2xz - 3yz = 40$

4 unknowns x, y, z, λ and 4 equations.