**Problem 1.** (25 pts) Compute the integral

$$\iint_D (x-y)dxdy,$$

where

$$D = \{(x,y) \mid -2 \le x \le 2,\ x^2 \le y \le 4\}.$$

$$\iint_D (x-y)dxdy = \int_{-2}^{2}\int_{x^2}^{4}(x-y)dydx = \int_{-2}^{2}(x(4-x^2)-(16-x^4)/2)dx$$
$$= -32 + 32/5 = -128/5.$$

**Problem 2.** (25 pts) Find the area of the region enclosed by the curve

$$r = \sin(2\theta), \quad 0 \le \theta \le \pi/2.$$

Denote the region by $D$. Then

$$D = \{(r, \theta) \,|\, 0 \le r \le \sin(2\theta),\ 0 \le \theta \le \pi/2\}.$$

Therefore,

$$\begin{aligned}\text{Area}(D) &= \iint_D dA = \iint_D dxdy = \int_0^{\pi/2} \int_0^{\sin(2\theta)} r dr d\theta \\ &= \int_0^{\pi/2} \frac{\sin^2(2\theta)}{2} d\theta = (1/4) \int_0^{\pi/2} (1 - \cos(4\theta)) d\theta = \pi/8.\end{aligned}$$

**Problem 3.** (25 pts) Find the center of mass of the region

$$R = \{(x,y)\,|\,0 \le x \le 1,\, 0 \le y \le 2\}$$

with the density

$$\rho(x,y) = x^2 + y^2.$$

Let $(\bar{x},\bar{y})$ be the center of mass. Then

$$m\bar{x} = M_y, \quad m\bar{y} = M_x,$$

where

$$m = \iint_R \rho(x,y)dA = \int_0^1 \int_0^2 (x^2+y^2)dydx = \int_0^1 (2x^2 + 8/3)dx = 10/3;$$

$$M_x = \iint_R y\rho(x,y)dA = \int_0^1 \int_0^2 (x^2y+y^3)dydx = \int_0^1 (2x^2 + 4)dx = 14/3;$$

$$M_y = \iint_R x\rho(x,y)dA = \int_0^1 \int_0^2 (x^3+xy^2)dydx = \int_0^1 (2x^3 + 8x/3)dx = 11/6.$$

Thus

$$(\bar{x},\bar{y}) = (M_y/m, M_x/m) = (11/20, 7/5).$$

**Problem 4.** (25 pts) Find the volume of the solid that lies between the paraboloid

$$2x^2 + 2y^2 = z$$

and the plane

$$z = 10.$$

Denote the solid by $S$. In cylindrical coordinates,

$$S = \{(r, \theta, z) \,|\, 0 \leq r \leq \sqrt{z/2},\ 0 \leq \theta \leq 2\pi,\ 0 \leq z \leq 10\}.$$

Hence

$$\begin{aligned}\text{Volume}(S) = \iiint_S dV &= \int_0^{10} \int_0^{2\pi} \int_0^{\sqrt{z/2}} r dr d\theta dz \\ &= (1/4) \int_0^{10} \int_0^{2\pi} z d\theta dz = (\pi/2) \int_0^{10} z dz = 25\pi.\end{aligned}$$