MATH 222 FINAL EXAM JULY 28, 2017

Show all your work in the space provided under each question. Please write neatly and present your answers in an organized way. You may use your one sheet of notes, but no books or calculators. This exam is worth 150 points.

Name:_____

1. (18 points) The vector field $\mathbf{F}(x,y) = \langle 3x^2y + x, x^3 + 3y^2 \rangle$ is conservative. Find a potential function $f(x,y)$.

2. (18 points) Find the curl and the divergence of the vector field $\mathbf{G} = \langle x^2y, y + z, z^3 \rangle$.

3. (18 points) Find the work done by a force described by the vector field $\mathbf{F}(x,y,z) = \langle x,-y,z\rangle$ moving a particle along the line segment from the point $(1,0,1)$ to the point $(2,3,4)$. (Assume the force is given in newtons and distances are given in meters.)

Cont.

4. (22 points) Use Green's theorem to compute the line integral

$$\oint_C xy\,dx + y^2 e^x\,dy$$

where C is the square with vertices $(0,0), (0,2), (2,2), (2,0)$.

5. (30 points) Evaluate $\iint_S y\,dS$ where S is the portion of the cylinder $x^2 + y^2 = 4$ that lies between $z = 0$ and $z = 6$ in the first octant.

Cont.

6. (24 points) Find the maximum and minimum values of the function $f(x,y) = x^2 + 2y^2$ on the closed unit disk $D = \{(x,y) | x^2 + y^2 \leq 1\}$.

7. (20 points) Consider the curve C parametrized by

$$\mathbf{r}(t) = \langle 3\sin(t), 3\cos(t), 4t \rangle$$

for $0 \leq t \leq 10$. Find the unit tangent vector $\mathbf{T}(t)$ of $\mathbf{r}(t)$ and the length of C.

The End.