Spring 2018 Math 222 Final

Name ____________________Instructor ____________________

No calculators. Not Books or notes. You have to show your ID to an instructor when you turn in your exam. You do not need to simplify arithmetic. Indeed, it is better to leave an answer like $\frac{2\cdot 15 - 6\cdot 4}{5^2+3^2}$.

1. Short answer questions.

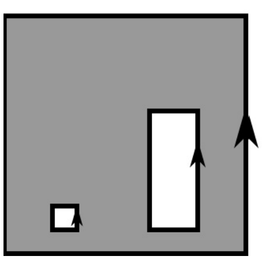
   (a) Compute $(\mathbf{i} \times \mathbf{k}) \cdot (\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$.

   (b) The following figure shows a 10×10 square, with a 1×1 square and a 2×5 rectangle inside. Let Γ be the boundary of the large square, γ be the boundary of the small square, and δ be the boundary of the medium rectangle (all oriented in the counter-clockwise directions as marked).
   Let $\nabla \times F = 2\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$. Further let

$$\int_\gamma F \cdot T ds = 5, \quad \text{and} \quad \int_\delta F \cdot T ds = 6.$$

   Find the value of

$$\int_\Gamma F \cdot T ds.$$



(c) Find a unit normal to the surface given by $z = x^2 + y^3$ at the point $(2, 1, 5)$.

(d) Find the equation of the line that passes through $(2, -1, 4)$ and is perpendicular to $3(x - 1) + 6(y - 5) + 7(z - 5) = 0$.

2. The divergence does not determine the vector field. The following questions address this issue.

   (a) Let $\nabla \cdot V = x^3 + 2xz$. It is reasonable to guess that there is a vector field V satisfying this that is fairly nice. In particular one of the components of V should be easy to guess. Which one? Explain your answer by listing a transformation of the form $(x, y, z) \mapsto (?, ?, ?)$.

   (b) Find a non-zero vector field V so that $\nabla \cdot V = 0$.

   (c) Find two different vector fields V and W so that $\nabla \cdot V = \nabla \cdot W = 8(x^2 + y^2)^3$. Express your answer using the variables (x, y, z) and unit vectors $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$.

3. Find the maximum and minimum values of $f(x, y) = x^2 + y$ subject to $x^2 + y^2 = 5/4$.

4. Use the substitution $x = u$, $y = u + 4v$ to simplify and evaluate

$$\int_{x=0}^{x=9} \int_{y=x}^{y=x+16} \frac{dy\,dx}{\sqrt{xy - x^2}}.$$

5. Let γ be the curve that starts at $(0, 0)$ and moves counter-clockwise around the triangle with corners $(0, 0)$, $(4, 0)$ and $(0, 4)$.

   Compute

$$\int_\gamma (y + e^{x^2})\, dx + (2x^2 - \sin(y^2))\, dy.$$