Instructions: Wait to open the exam until instructed to do so. Then answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page. You will have 1 hour to complete this exam.

Question	Points	Score
1	30	
2	20	
3	10	
4	10	
5	15	
6	15	
Total:	100	

Name: ______________________________

Recitation Instructor: ______________________________

Recitation Time: ______________________________

1. For the following questions, suppose $\mathbf{u} = \langle 1, 1, 1 \rangle$ and $\mathbf{v} = \langle 0, -1, 0 \rangle$.

   (a) (5 points) Evaluate $2\mathbf{u} + \mathbf{v}$.

   (b) (5 points) Evaluate $\mathbf{u} \cdot \mathbf{v}$.

   (c) (5 points) Find $\cos(\theta)$ where θ is the angle between $\mathbf{u}$ and $\mathbf{v}$.

(d) (5 points) Evaluate $\mathbf{u} \times \mathbf{v}$.

(e) (5 points) Find the area of the parallelogram spanned by $\mathbf{u}$ and $\mathbf{v}$.

(f) (5 points) Find the distance between the point $(1, 0, 0)$ and the plane parallel to $\mathbf{u}$ and $\mathbf{v}$ which passes through the origin.

2. Solve the problems regarding the points $P = (1, 1, -1)$, $Q = (1, 1, 1)$ and the vector $\mathbf{v} = \langle 2, 1, 1 \rangle$.

   (a) (5 points) Find a parametric equation for the line ℓ passing through P with direction vector $\mathbf{v}$.

   (b) (5 points) Is Q on the line found in part (a)? Explain your response.

(c) (5 points) Find the equation for the plane passing through Q and with normal vector $\mathbf{v}$.

(d) (5 points) Give an equation for a line that passes through P and is parallel to the plane found in part (c).

3. (a) (5 points) Sketch and describe the trace of the intersection of the plane $z = -3$ with the surface $x^2 + y^2 - z^2 = 16$.

(b) (5 points) Give the inequalities in spherical coordinates that describe the upper unit half ball, which in Cartesian coordinates is described by:

$$z \geq 0, \qquad x^2 + y^2 + z^2 \leq 1$$

4. (10 points) Given the vectors $\mathbf{u} = \langle 2, 1, 0 \rangle$, $\mathbf{v} = \langle 0, -1, 1 \rangle$ and the parameterized line $\mathbf{r}(t) = \langle t, 1 - t, 2t \rangle$, find a11 values of t for which the area of the parallelogram spanned by $\mathbf{u}$ and $\mathbf{v}$ is equal to the volume of the parallelepiped spanned by $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{r}(t)$ (ignoring units).

5. Answer the following questions concerning the vector valued function

$$\mathbf{r}(t) = \left\langle \ln(t), t^2, e^{t-1} \right\rangle$$

for $t > 0$.

(a) (5 points) Evaluate

$$\lim_{t \to 1} \mathbf{r}(t).$$

(b) (5 points) Evaluate $\mathbf{r}'(t)$.

(c) (5 points) Evaluate

$$\int_1^2 \mathbf{r}(t)\, \mathrm{d}t.$$

6. Consider the vector valued function

$$\mathbf{r}(t) = \langle \sin(4t), \cos(4t), 3t \rangle$$

for $0 \leq t \leq 4$.

(a) (5 points) Find the arc-length function $s(t)$ of $\mathbf{r}(t)$.

(b) (5 points) What is the length of the curve parametrized by $\mathbf{r}(t)$?

(c) (5 points) Find the arc-length parametrization $\mathbf{r}(s)$.

Some formulas

$$\mathbf{u} = \langle u_1, u_2, u_3 \rangle$$
$$\mathbf{v} = \langle v_1, v_2, v_3 \rangle$$
$$\mathbf{u} \times \mathbf{v} = \langle u_2v_3 - v_2u_3, u_3v_1 - u_1v_3, u_1v_2 - u_2v_1 \rangle$$

$$\text{proj}_{\mathbf{u}}\mathbf{v} = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\|^2}\mathbf{u}$$

Coordinate systems

Cylindrical

$$x = r\cos(\theta)$$
$$y = r\sin(\theta)$$
$$z = z$$

$$r = \sqrt{x^2 + y^2}$$
$$\tan(\theta) = \frac{y}{x}$$
$$z = z$$

Spherical

$$x = \rho\cos(\theta)\sin(\varphi)$$
$$y = \rho\sin(\theta)\sin(\varphi)$$
$$z = \rho\cos(\varphi)$$

$$\rho = \sqrt{x^2 + y^2 + z^2}$$
$$\tan(\theta) = \frac{y}{x}$$
$$\cos(\varphi) = \frac{z}{\rho}$$