**Instructions**: Wait to open the exam until instructed to do so. Then answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page. You will have 1 hour to complete this exam.

| Question | Points | Score |
|---|---|---|
| 1 | 15 | |
| 2 | 15 | |
| 3 | 10 | |
| 4 | 15 | |
| 5 | 15 | |
| 6 | 15 | |
| 7 | 15 | |
| Total: | 100 | |

**Name:** ______________________________

**Recitation Instructor:** ______________________________

**Recitation Time:** ______________________________

1. Find the limit, if it exists. If the limit does not exist, explain why.

   (a) (5 points)

$$\lim_{(x,y)\to(0,0)} \frac{x-y}{x^2+y^2}$$

   (b) (5 points)

$$\lim_{(x,y)\to(0,0)} e^{-\frac{1}{x^2+y^2}}$$

   (c) (5 points)

$$\lim_{(x,y)\to(0,0)} \frac{xy^3}{x^2+y^2}$$

2. Evaluate the partial derivatives, if they exist. If they do not exist, explain why.

(a) (5 points) $f_y(1, -2)$ for $f(x, y) = xy + \sin(\pi xy)$.

(b) (5 points) $\frac{\partial^3 f}{\partial y \partial x^2}(0, 0)$ for $f(x, y) = x^3 + x^2 + x + 1 + y^3 + y^2 + x^2 y$

(c) (5 points) $f_z(0, 1, 0)$ for

$$f(x, y, z) = 2y|x| + 3x|z| + 4z|y| + 5z.$$

3. Consider the function $f(x,y) = e^{xy} + 3x$.

(a) (5 points) Give the linearization $L(x,y)$ of $f(x,y)$ at $(2,0)$.

(b) (5 points) Use your result from part (a) to write the equation of the tangent plane to the graph of $f(x,y)$ at $(2,0)$.

4. Let

$$f(x, y, z) = x^2 + 4y^2 - z^2$$

(a) (5 points) Find the gradient of $f$.

(b) (5 points) Which points on the level surface

$$f(x, y, z) = 1$$

have tangent planes parallel to the plane $y = 0$.

(c) (5 points) Give an example of a vector $\mathbf{v}$ for which $f$ is decreasing in the direction of $\mathbf{v}$ starting at $(3, 2, 1)$.

5. (15 points) Let

$$f(r, \theta, z) = r^2 + \sin(\theta)$$

be written in cylindrical coordinates. Use the chain rule to calculate $\frac{\partial f}{\partial x}$ at the point $P$ which is $(x, y, z) = (-1, 0, 1)$ in Cartesian coordinates. Confirm your result by expressing $f$ in Cartesian coordinates and taking its partial derivative with respect to $x$.

6. Consider the function

$$f(x, y) = x^3 - 3x - y^3 + 3y$$

on the rectangular domain $\mathcal{D}$ defined by

$$-\frac{3}{2} \leq x \leq \frac{3}{2}, \qquad -\frac{3}{2} \leq y \leq 0$$

(a) (5 points) Find the critical points of $f$ in the interior of $\mathcal{D}$.

(b) (5 points) Describe the local behavior of $f$ near the critical points.

(c) (5 points) The global maximum and minimum values of $f$ on the boundary of the rectangle are 2 and $-\frac{25}{8}$ respectively. Find the global maximum value and the global minimum value for $f$ on $\mathcal{D}$ if they exist. Explain your response.

7. (15 points) Use Lagrange multipliers to find the critical points of the function

$$f(x, y, z) = 2x + 2y - z$$

on the sphere

$$x^2 + y^2 + z^2 = 4.$$

Identify the global maximum and minimum values of $f$ on the sphere.

## Coordinate systems

Cylindrical

$$x = r\cos(\theta)$$
$$y = r\sin(\theta)$$
$$z = z$$

$$r = \sqrt{x^2 + y^2}$$
$$\tan(\theta) = \frac{y}{x}$$
$$z = z$$

Spherical

$$x = \rho\cos(\theta)\sin(\phi)$$
$$y = \rho\sin(\theta)\sin(\phi)$$
$$z = \rho\cos(\phi)$$

$$\rho = \sqrt{x^2 + y^2 + z^2}$$
$$\tan(\theta) = \frac{y}{x}$$
$$\cos(\phi) = \frac{z}{\rho}$$

## Derivative formulas

Directional derivative :

$$D_{\mathbf{u}}f(x, y) = \nabla f(x, y) \cdot \mathbf{u}$$

Chain rule, general case for $f(x_1, \ldots, x_m)$ and $x_j(t_1, \ldots, t_n)$:

$$\frac{\partial f}{\partial t_i} = \frac{\partial f}{\partial x_1}\frac{\partial x_1}{\partial t_i} + \cdots + \frac{\partial f}{\partial x_m}\frac{\partial x_m}{\partial t_i}$$

Derivative formula

$$\frac{d}{du}\tan^{-1} u = \frac{1}{1 + u^2}$$