**Instructions**: Wait to open the exam until instructed to do so. Then answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page. You will have 1 hour to complete this exam.

| Question | Points | Score |
|---|---|---|
| 1 | 15 | |
| 2 | 15 | |
| 3 | 15 | |
| 4 | 20 | |
| 5 | 15 | |
| 6 | 10 | |
| 7 | 10 | |
| Total: | 100 | |

**Name:** ______________________________

**Recitation Instructor:** ______________________________

**Recitation Time:** ______________________________

1. (15 points) Calculate the integral

$$\iiint_{\mathcal{B}} x^2 y \cos(xyz) \, \mathrm{d}V$$

where $\mathcal{B} = [0, \pi] \times [0, 1] \times [-1, 0]$.

2. (15 points) Calculate the integral of

$$f(x, y) = (1 - x)^2$$

over the region

$$\mathcal{D} : 0 \leq x \leq 1 - y^2, 0 \leq y$$

3. Consider the region

$$\mathcal{W} : x^2 + y^2 + z^2 \leq 25,\ x^2 + y^2 \geq 16$$

(a) (10 points) Express the volume of $\mathcal{W}$ as an iterated integral using cylindrical coordinates.

(b) (5 points) Evaluate the integral to obtain $\mathrm{Vol}(\mathcal{W})$.

4. Let $\mathcal{D}$ be the parallelogram in the plane with vertices $(0,0), (1,0), (2,3), (3,3)$.

   (a) (10 points) Find a linear mapping $G(u,v)$ which sends the unit square $[0,1] \times [0,1]$ to $\mathcal{D}$.

   (b) (5 points) Compute the Jacobian of $G$.

(c) (5 points) Use the change of variables formula to compute the integral

$$\iint_{\mathcal{D}} e^{3x-2y}\, \mathrm{d}A$$

5. (15 points) Calculate

$$\int_C e^{x^2+y^2+z^2}\, ds$$

where $C$ is the equator of a sphere, centered at the origin, of radius 3.

6. (10 points) Evaluate

$$\int_C \mathbf{F} \cdot d\mathbf{r}$$

where $\mathbf{F}(x, y, z) = \langle z, xyz, x \rangle$ and $C$ is the curve parameterized by

$$\mathbf{r}(t) = (e^t, t, e^{-t})$$

for $0 \leq t \leq 2$.

7. Consider the vector field

$$\mathbf{F}(x, y, z) = \langle y + z, x + z, x + y \rangle$$

(a) (5 points) Does $\mathbf{F}$ satisfy the cross-partials condition? Verify your response.

(b) (5 points) Find a potential for $\mathbf{F}$ if one exists. If not, explain why.

**Coordinate systems**

| Polar | Cylindrical | Spherical |
|---|---|---|
| $x = r\cos(\theta)$ | $x = r\cos(\theta)$ | $x = \rho\cos(\theta)\sin(\phi)$ |
| $y = r\sin(\theta)$ | $y = r\sin(\theta)$ | $y = \rho\sin(\theta)\sin(\phi)$ |
| | $z = z$ | $z = \rho\cos(\phi)$ |
| $r = \sqrt{x^2+y^2}$ | $r = \sqrt{x^2+y^2}$ | $\rho = \sqrt{x^2+y^2+z^2}$ |
| $\tan(\theta) = \dfrac{y}{x}$ | $\tan(\theta) = \dfrac{y}{x}$ | $\tan(\theta) = \dfrac{y}{x}$ |
| | $z = z$ | $\cot(\phi) = \dfrac{z}{\sqrt{x^2+y^2}}$ |
| $\mathrm{d}x\,\mathrm{d}y = r\,\mathrm{d}r\,\mathrm{d}\theta$ | $\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z = r\,\mathrm{d}r\,\mathrm{d}\theta\,\mathrm{d}z$ | $\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z = \rho^2\sin\phi\,\mathrm{d}\rho\,\mathrm{d}\phi\,\mathrm{d}\theta$ |

**Change of variables**

$$G : \mathcal{D}_0 \to \mathcal{D}$$

$$G(u,v) = (x(u,v), y(u,v))$$

$$\mathrm{Jac}(G) = \frac{\partial x}{\partial u}\frac{\partial y}{\partial v} - \frac{\partial x}{\partial v}\frac{\partial y}{\partial u}$$

$$\iint_{\mathcal{D}} f(x,y)\,\mathrm{d}x\,\mathrm{d}y = \iint_{\mathcal{D}_0} f(x(u,v), y(u,v))\,|\mathrm{Jac}(G)|\,\mathrm{d}u\,\mathrm{d}v$$

**Line integrals**

$\mathbf{r}(t)$ for $a \le t \le b$ parametrizing $\mathcal{C}$

$$\int_{\mathcal{C}} f(x,y,z)\,\mathrm{d}s = \int_a^b f(\mathbf{r}(t))\,\|\mathbf{r}'(t)\|\,\mathrm{d}t$$

$$\int_{\mathcal{C}} \mathbf{F}\cdot\mathrm{d}\mathbf{r} = \int_a^b \mathbf{F}(\mathbf{r}(t))\cdot\mathbf{r}'(t)\,\mathrm{d}t$$